Hyperbolic functions and hyperbolic identities

Objectives:

- ◇ Learn names and definitions of the hyperbolic (trigonometric) functions
- ◇ Be able to evaluate the hyperbolic functions at given angles
- ◇ Be able to use identities involving hyperbolic functions

Key points:

These new functions have **similar properties** in equations to the standard trigonometry functions, however they are different functions. Rather than sin and cos the names to remember are:

- ◇ sinh pronounced 'shine' or 'sinch'
- ◇ cosh pronounced 'cosh' as you'd expect
- ◇ tanh pronounced 'th-an' like thank without the k, or 'tanch' (this is just sinh $\div$ cosh)

They are defined using the special number e (Euler's constant) ≈ 2.71828. Some people just use the e^x notation below, others call it the exponential function and define $\exp(x) \equiv e^x$.

To evaluate $\sinh(0.4)$, for example, involves substituting $x = 0.4$ into the formula for $\sinh(x)$:

$$\sinh(x) = \frac{1}{2}\left(e^x - e^{-x}\right) \quad \text{so } \sinh(0.4) = \frac{1}{2}\left(e^{0.4} - e^{-0.4}\right) \approx \frac{1}{2}(1.492 - 0.670) = 0.411$$

There are **hyperbolic identities** just like there were trigonometric identities and they are almost the same! Annoyingly, though, they aren't quite the same. There is no easy rule, but the main difference is that equations that used $\sin^2(x)$ now use $-\sinh^2(x)$, **note the minus sign**. So,

$\boxed{\cos^2(x) + \sin^2(x) = 1}$ has a matching identity $\boxed{\cosh^2(x) - \sinh^2(x) = 1}$ and

$\boxed{\sin(2x) = 2\sin(x)\cos(x)}$ matches with $\boxed{\sinh(2x) = 2\sinh(x)\cosh(x)}$.

Also noteworthy: $\sinh(x) = -\sinh(-x)$ and $\cosh(x) = -\cosh(x)$ just like with sin and cos.

Recommended links:

Highly recommended: HELM notes (Hyperbolic Functions)

Highly recommended: Mathcentre handout (Definitions), Mathcentre handout (Identities)

Other links: Mathcentre text, YouTube Khan Academy video