Integration: indefinite and definite integrals

Objectives:

- Appreciate that integration can be seen as both anti-differentiation, and also a method for finding areas under curves
- Learn what definite and indefinite integrals are, and the difference between them

Key points:

Integrating is another procedure, like differentiating, that you can do to a function. The function you try and integrate, let's call it f, is called the **integrand** and the result from integrating f is called the **integral of f**.

A **definite integral** is written like this:

$$\int_1^3 x^2+1\,\mathrm{d}x$$

An **indefinite integral** is written like this:

$$\int x^2+1\,\mathrm{d}x$$

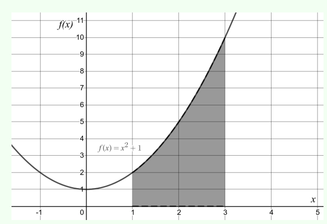
There are three parts...

- the $\int_a^b$ part which tells you to integrate and if there are limits what they are. The definite integral says to **integrate from a to b** (here it says from 1 to 3)
- the integrand/function to integrate, x^2+1 in both the examples here
- the $\mathrm{d}x$ part which tells you the variable to use for the integration

As anti-differentiation, since $g(x)=\frac{1}{3}x^3+x+C$ for any constant C has derivative $g'(x)=x^2+1$ then we say g is the indefinite integral of $f(x)=x^2+1$. It's any function whose derivative is f.

To find an area under a curve, we calculate the definite integral between the two limits stated. In this case, between $x=1$ and $x=3$. Notationally we write

$$\int_1^3 x^2+1\,\mathrm{d}x=\left[\frac{1}{3}x^3-x+C\right]_1^3=[g(x)]_1^3=g(3)-g(1)=\frac{32}{3}$$



Recommended links:

Highly recommended: HELM notes (Indefinite integrals: the basics, rules and table of standard results), HELM notes (Definite integrals: basics and engineering applications)

Recommended: Mathcentre handout (Explanation of integration as reverse of differentiation), Mathcentre handout (Explanation of integration as a summation, with engineering example), Mathcentre handout (Summary on definite integration) see also Mathcentre notes (Finding areas with integration)