Answers to Exercises in Chapter 3

3.1 Suitable equations are:

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} \text{ or } Q = \int I \mathrm{d}t$$

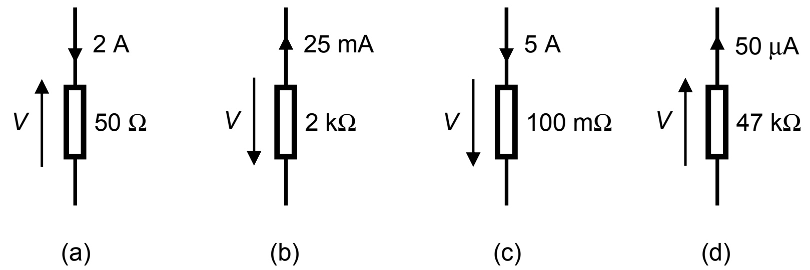
3.2 I is constant so $Q = I \times t = 5 \times 10 = 50$ coulombs.

3.3 Zero Ohms.

3.4 A voltage source where the output voltage is controlled by an external quantity.

3.5 An ideal current source has an infinite output resistance.

3.6



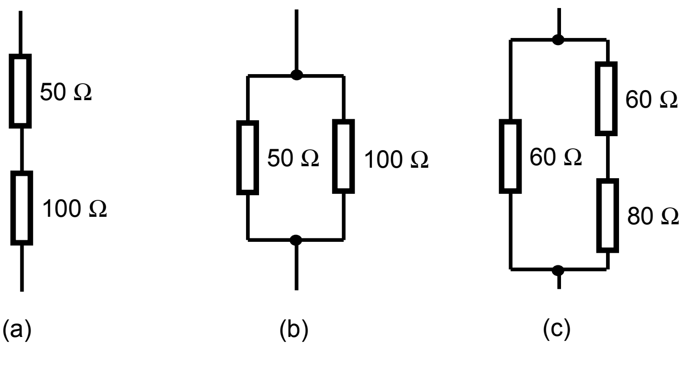
(a) V = 100 V; (b) 50 V; (c) −0.5 V; (d) −2.35 V.

3.7 (a) $P = I^2R$ = 200 W; (b) P = 1.25 W; (c) P = 2.5 W; (d) P = 117.5 μW.

3.8 $R = \rho l/A = 1.6 \times 10^{-8} \times 1/10^{-6} = 16\ \mathrm{m}\Omega$.

3.9 The conductivity of a material (σ) is equal to the reciprocal of the resistivity (ρ). Therefore $\sigma = 1/\rho$.

3.10



(a) $R = 50 + 100 = 150\ \Omega$;

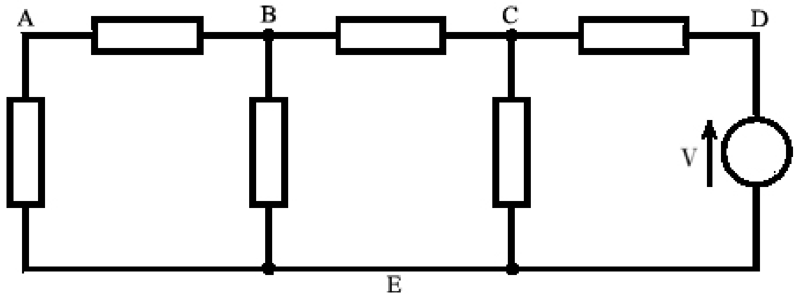
(b) $R = 1/(1/50 + 1/100) = 33.3\ \Omega$;

(c) $R = 1/(1/60 + 1/(60 + 80)) = 42\ \Omega$.

3.11 $R = 1/(\ 1/R_1 + 1/R_2) = 1/(1/10\ \text{k}\Omega + 1/10\ \text{k}\Omega) = 5\ \text{k}\Omega$.

3.12 A point within a circuit where two or more circuit components are joined together is termed a *node*, while any closed path within a circuit that passes through no node more than once is termed a *loop*. A loop that contains no other loop is called a *mesh*.

3.13 The labelling of the nodes is arbitrary. A suitable scheme might be as follows:

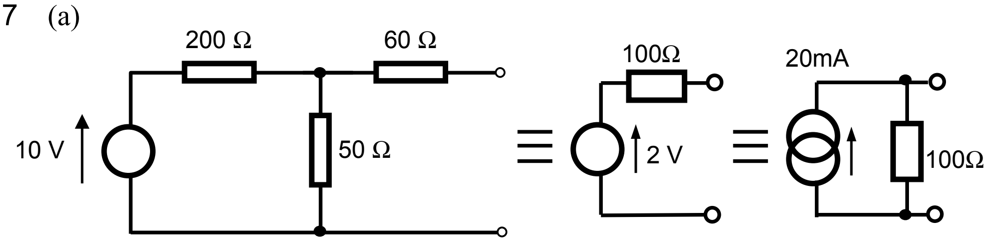


3.14 Using the notation shown above, the loops within the circuit are: ABEA, BCEB, CDEC, ABCEA, BCDEB and ABCDEA. The meshes are ABEA, BCEB and CDEC.

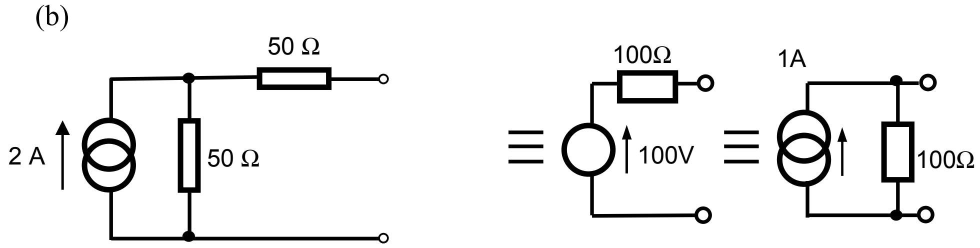
3.15 Summing the currents flowing *into* the node and equating this to zero gives: $-I_1 - I_2 + I_3 + I_4 = 0$. Therefore, $-6\text{A} - I_2 + 9\text{A} + 5\text{A} = 0$, and thus $I_2 = 8\text{A}$.

3.16 Summing the voltages clockwise around the mesh and equating this to zero gives: $-V_1 + V_2 - E + V_3 = 0$. Therefore, $-2\text{V} + V_2 - 12\text{V} + 4\text{V} = 0$, and thus $V_2 = 10\text{V}$.

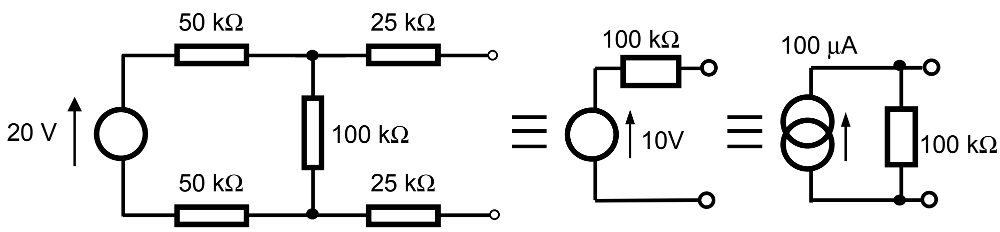
3.17 (a)



(b)



(c)



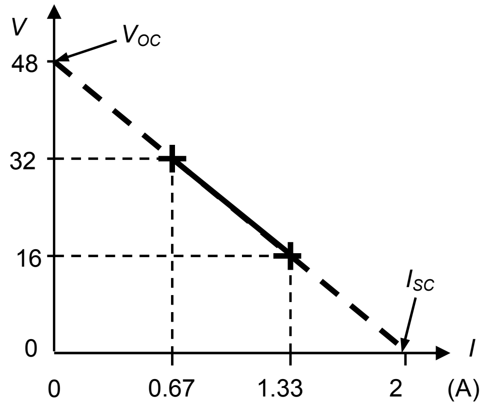
3.18

When R_L = 12 Ω, V_o = 16 V,
so I =1.33A

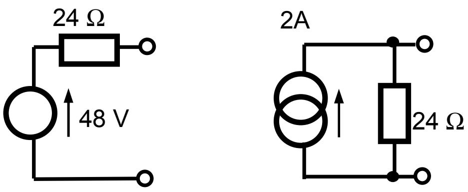
When R_L = 48 Ω, V_o = 32 V,
so I =0.67A

Plotting a graph (as right) gives V_{oc} = 48 V and I_{SC} =2A.

Therefore, $R = V/I$ = 48/2 = 24 Ω.

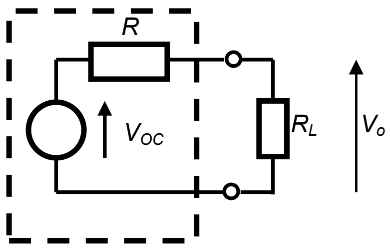


Therefore, the Thévenin and Norton equivalent circuits of this arrangement are:



3.19 An analytical solution to the problem of Exercise 3.18 is as follows:

Consider a Thévenin equivalent circuit of the unknown network plus the load resistors. This is:



For this arrangement

$$V_o = V_{oc}\frac{R_L}{R_L + R}$$

When $R_L = 12\ \Omega$, $V_o = 16$ V so

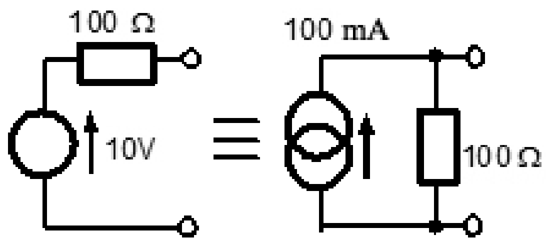
$$16 = V_{oc}\frac{12}{12 + R}$$
$$192 + 16R = 12V_{oc}$$

When $R_L = 48\ \Omega$, $V_o = 32$ V so

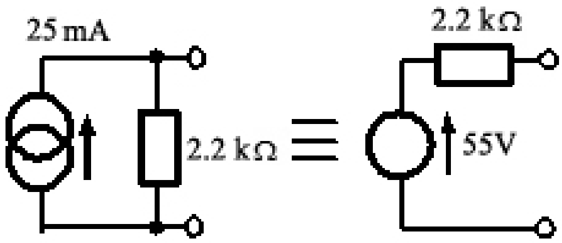
$$32 = V_{oc}\frac{48}{48 + R}$$
$$1536 + 48R = 48V_{oc}$$

Solving these two simultaneous equations gives $V_{oc} = 48$ V and $R = 24\ \Omega$, as before, and $I_{sc} = V_{oc}/R = 48/2 = 24\ \Omega$. Hence, the Thévenin and Norton equivalent circuits are as before.

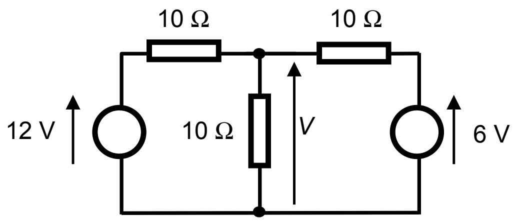
3.20 The short circuit current of a Thévenin equivalent circuit is related to the open-circuit voltage and the resistance by the expression: $R = V_{OC}/I_{SC}$. Therefore, $I_{SC} = V_{OC}/R$, which in this case gives $I_{SC} = 10\text{V}/100\Omega = 100\text{mA}$. Therefore, an appropriate equivalent circuit is as shown here:



3.21 The open-circuit circuit voltage of a Norton equivalent circuit is related to the short-circuit current and the resistance by the expression: $R = V_{OC}/I_{SC}$. Therefore, $V_{OC} = R \times I_{SC}$, which in this case gives $V_{OC} = 2.2\text{k}\Omega \times 25\text{mA} = 55\text{V}$. Therefore, an appropriate equivalent circuit is as shown here:



3.22 (a)

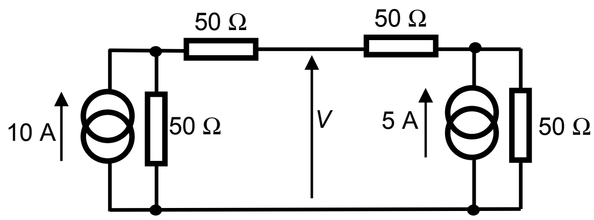


With just the 12 V source (and the 6 V source replaced by a short circuit) $V = 12 \times ((10//10)/((10//10) + 10)) = 4$ V.

With just the 6 V source (and the 12 V source replaced by a short circuit) $V = 6 \times ((10//10)/((10//10) + 10)) = 2$ V.

Therefore, with both sources present the voltage $V = 4 + 2 = 6$V.

(b)

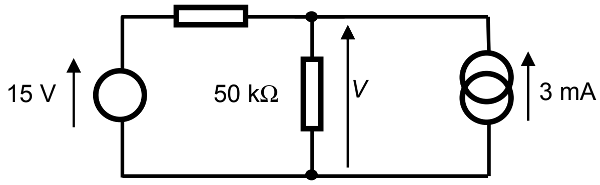


With just the 10 A source (and the 5 A source replaced by an open circuit) $V = 10 \times (50//150) \times (100/150) = 10 \times 37.5 \times 2/3 = 250$ V.

With just the 5 A source (and the 10 A source replaced by an open circuit) $V = 5 \times (50//150) \times (100/150) = 5 \times 37.5 \times 2/3 = 125$ V.

Therefore, with both sources present the voltage $V = 250 + 125 = 375$ V.

(c)

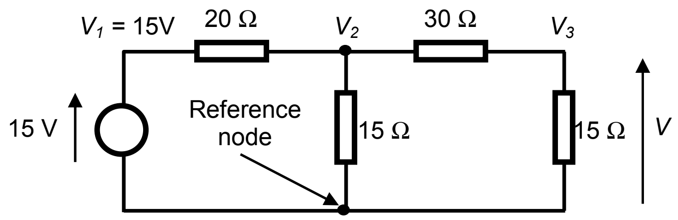


With just the 15V source (and the 3 mA source replaced by an open circuit) $V = 15 \times (50\text{ k}\Omega/(50\text{ k}\Omega + 25\text{ k}\Omega))$ 10 V.

With just the 3 mA source (and the 15 V source replaced by a closed circuit) $V = 3\text{ mA} \times (50\text{ k}\Omega\ //\ 25\text{ k}\Omega\ ) = 3\text{ mA} \times 16.7\text{ k}\Omega = 50\text{ V}$.

Therefore, with both sources present the voltage $V = 10 + 50 = 60$ V.

3.23



First, we pick our reference node, and label the various node voltages, as shown above. Next, we sum the currents flowing into the nodes for which the node voltages are unknown. This gives

$$\frac{15-V_2}{20}+\frac{V_3-V_2}{30}+\frac{0-V_2}{15}=0$$

and

$$\frac{V_2-V_3}{30}+\frac{0-V_3}{15}=0$$

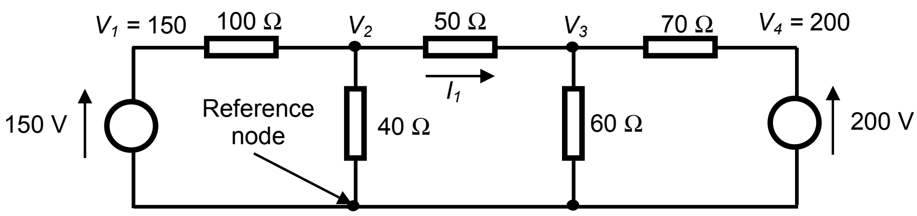
Solving these two equations, (which is left as an exercise for the reader) gives

$$V_2 = 5.4\text{ V}$$

$$V_3 = 1.8\text{ V}$$

In this case, the output voltage $V = V_3 = 1.8$ V.

3.24



First, we pick our reference node, and label the various node voltages, as shown above. Next, we sum the currents flowing into the nodes for which the node voltages are unknown. This gives

$$\frac{150-V_2}{100}+\frac{V_3-V_2}{50}+\frac{0-V_2}{40}=0$$

and

$$\frac{V_2-V_3}{50}+\frac{200-V_3}{70}+\frac{0-V_3}{60}=0$$

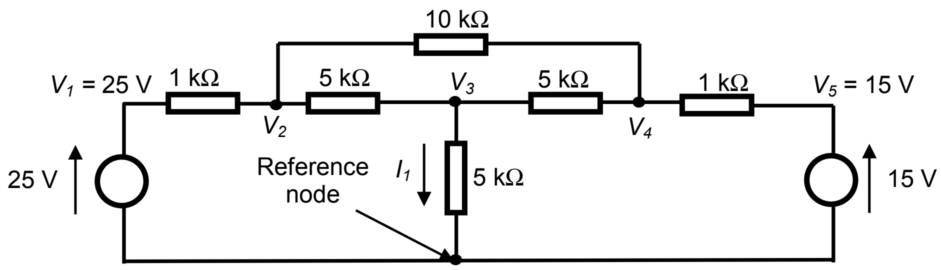
Solving these two equations, (which is left as an exercise for the reader) gives

$$V_2 = 55.6 \text{ V}$$

$$V_3 = 77.9 \text{ V}$$

The current I_1 is then given by $I_1 = (V_2 - V_3)/50 = (55.6 - 77.9)/50 = -446$ mA.

3.25



First, we pick our reference node, and label the various node voltages, as shown above. Next we sum the currents flowing into the nodes for which the node voltages are unknown. This gives

$$\frac{25-V_2}{1\,\text{k}\Omega}+\frac{V_3-V_2}{5\,\text{k}\Omega}+\frac{V_4-V_2}{10\,\text{k}\Omega}=0$$

$$\frac{V_2-V_3}{5\,\text{k}\Omega}+\frac{V_4-V_3}{5\,\text{k}\Omega}+\frac{0-V_3}{5\,\text{k}\Omega}=0$$

and

$$\frac{V_3-V_4}{5\,\text{k}\Omega}+\frac{V_2-V_4}{10\,\text{k}\Omega}+\frac{15-V_4}{1\,\text{k}\Omega}=0$$

Solving these three equations (which is left as an exercise for the reader) gives

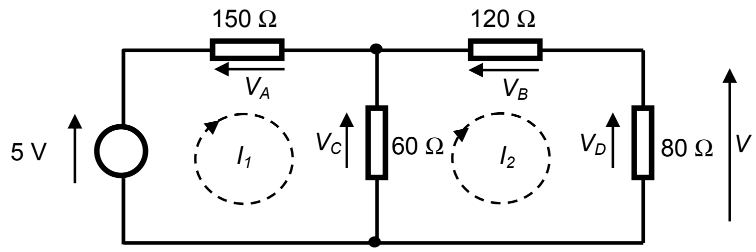
$$V_2 = 22.32 \text{ V}$$

$$V_3 = 12.5 \text{ V}$$

$$V_4 = 15.18 \text{ V}$$

The current I_1 is then given by $I_1 = V_3/5$ kΩ = 12.5/5 kΩ = 2.5 mA.

3.26



The circuit contains two meshes. To these, we assign loop currents I_1 and I_2 as shown above. The diagram also defines the various voltages.

The next stage is to apply Kirchhoff's voltage law to each mesh, which gives the following simultaneous equations.

$$5 - 150I_1 - 60(I_1 - I_2) = 0$$
$$60(I_1 - I_2) - 120I_2 - 80I_2 = 0$$

These simultaneous equations may be solved to give

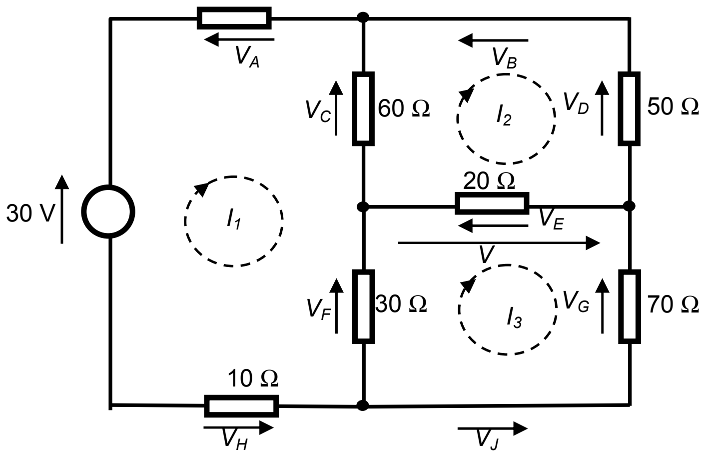
$$I_1 = 25.5 \text{ mA}$$
$$I_2 = 5.88 \text{ mA}$$

The voltage across the 80 Ω resistor is the product of its resistance and the current through it, so

$$\begin{aligned} V &= 80I_2 \\ &= 80\,\Omega \times 5.88 \text{ mA} \\ &= 0.47 \text{ V} \end{aligned}$$

As the calculated voltage is positive, the polarity is as shown by the arrow with the left hand end of the resistor more positive than the right hand end.

3.27



The circuit contains three meshes. To these, we assign loop currents I_1, I_2 and I_3, as shown above. The diagram also defines the various voltages.

The next stage is to apply Kirchhoff's voltage law to each mesh, which gives the following simultaneous equations.

$$
\begin{aligned}
30 - 40I_1 - 60(I_1 - I_2) - 30(I_1 - I_3) - 10I_1 &= 0 \\
60(I_1 - I_2) - 50I_2 + 20(I_3 - I_2) &= 0 \\
30(I_1 - I_3) - 20(I_3 - I_2) - 70I_3 &= 0
\end{aligned}
$$

These simultaneous equations may be solved to give

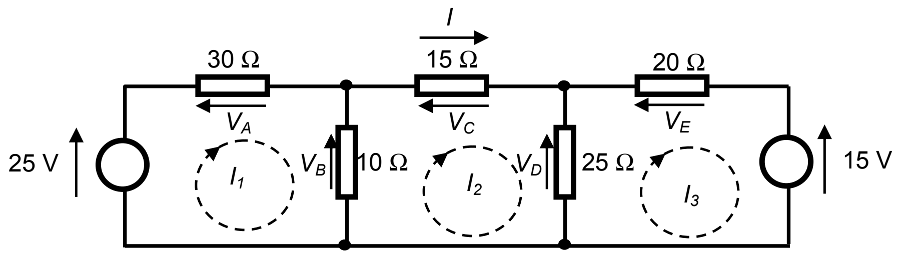
$$
\begin{aligned}
I_1 &= 302\ \text{mA} \\
I_2 &= 155\ \text{mA} \\
I_3 &= 101\ \text{mA}
\end{aligned}
$$

The required voltage V is equal to $-V_{E,}$ which is given by

$$
\begin{aligned}
V = -V_E &= 20(I_2 - I_3) \\
&= 20(0.155 - 0.101) \\
&= 1.08\ \text{V}
\end{aligned}
$$

As the calculated voltage is positive, the polarity is as shown by the arrow.

3.28



The circuit contains three meshes. To these, we assign loop currents I_1, I_2 and I_3, as shown above. The diagram also defines the various voltages.

The next stage is to apply Kirchhoff's voltage law to each mesh, which gives the following simultaneous equations.

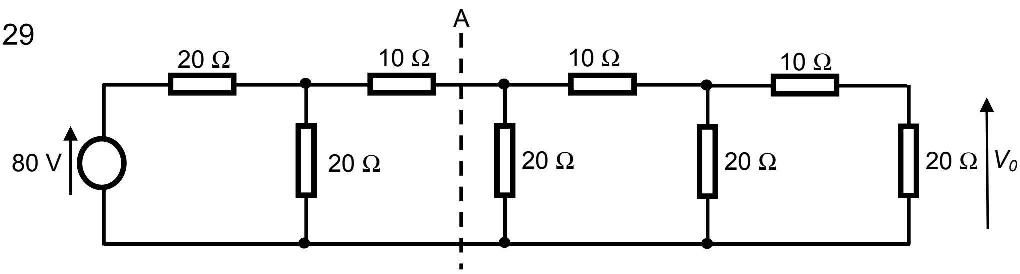
$$25-30I_1-10(I_1-I_2)=0$$
$$10(I_1-I_2)-15I_2-25(I_2-I_3)=0$$
$$25(I_2-I_3)-20I_3-15=0$$

The three simultaneous equations may be solved to give

$$I_1 = 610 \text{ mA}$$
$$I_2 = -62 \text{ mA}$$
$$I_3 = -368 \text{ mA}$$

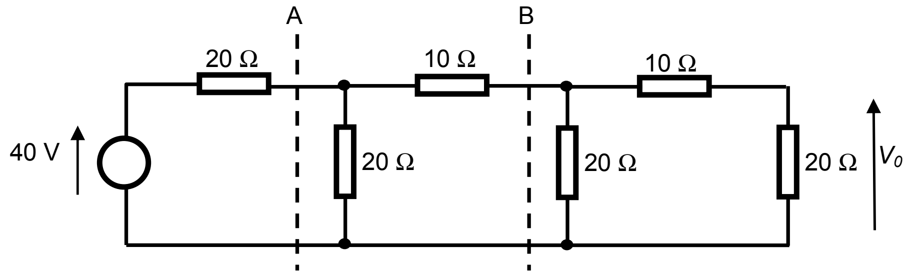
The required current is equal to I_2.

3.29

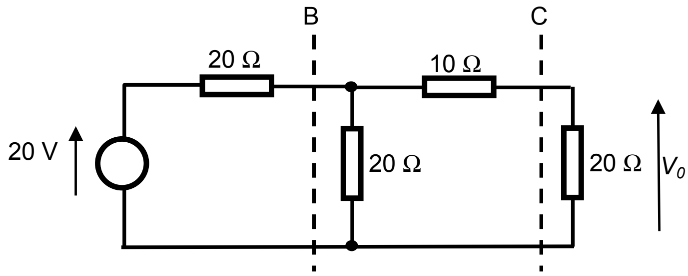


This circuit can be analysed in many ways. One attractive method is to use Thévenin's theorem to replace parts of the circuit with simpler arrangements.

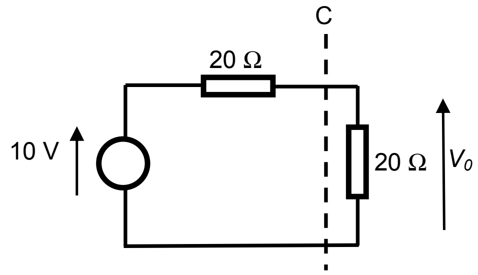
For example, the circuit to the left of the dotted line 'A' can be replaced by its Thévenin equivalent circuit giving:



The circuit to the left of the dotted line 'B' can then be replaced by its Thévenin equivalent circuit giving:



The circuit to the left of the dotted line 'C' can then be replaced by its Thévenin equivalent circuit giving:



From which it is clear that $V_0 = 5$ V.