Answers to Exercises in Chapter 4

4.1 A dielectric is the insulating layer between the two plates of a capacitor.

4.2 A positive charge takes the form of a deficit of negatively charged electrons.

4.3 When a charge is applied to a capacitor, electrons flow onto one plate and away from the other. As electrons flow *into* one side of the capacitor and flow *out of* the other, it might appear that current is flowing *through* it.

4.4 If a resistor is connected across a charged capacitor, the stored charge will drive a current through the resistor, discharging the capacitor, and releasing the stored energy. The capacitor therefore acts a little like a 'rechargeable battery'.

4.5 The relationship between these two quantities is given by the capacitance C of the capacitor, such that

$$C = \frac{Q}{V}$$

If the charge is measured in *coulombs* and the voltage in *volts*, then the capacitance has the units of **Farads**.

4.6 Farads.

4.7 $Q = CV = 100\ \mu\text{F} \times 5\text{V} = 500\ \mu\text{C}$.

4.8 $V = Q/C = 1\ \text{mC}/22\ \mu\text{F} = 45.5\ \text{V}$.

4.9 $C = Q/V = 500\ \mu\text{C} / 25\ \text{V} = 20\ \mu\text{F}$.

4.10 This is discussed in Section 4.3.

4.11

$$C = \frac{\varepsilon A}{d}$$

Where ε is the permittivity of the dielectric, A is the area and d is the separation of the plates. ε is sometimes represented by the product $\varepsilon_0\varepsilon_r$ where ε_0 is the absolute permittivity and ε_r is the relative permittivity of the dielectric used.

4.12 The permittivity of air is about 8.85 picofarads per metre, so $C = \varepsilon A/d = 8.85 \times 10^{-12} \times 5 \times 10^{-3} \times 15 \times 10^{-3}/10 \times 10^{-6} = 66$ pF.

4.13 This would increase the capacitance by a factor of 200 to 200 × 66 pF = 13.2 nF.

4.14 A small amount of capacitance exists between each of the conductors within electrical circuits and between the various elements within electrical components. These small, unintended capacitances are called **stray capacitances** and can have a very marked effect on circuit behaviour. The need to charge and discharge these stray capacitances limits the speed of operation of circuits and is a particular problem in high-speed circuits and those that use small signal currents.

4.15 When charged particles experience a force as a result of their charge, we say that an electric field exists in that region. The magnitude of the force exerted on a charged particle is determined by the electric field strength, E, at that point in space.

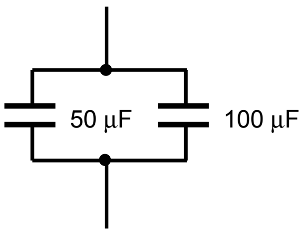
4.16 $E = V/d = 250/15 \times 10^{-6} \approx 17$ MV/m.

4.17 All insulating materials have a maximum value for the field strength that they can withstand before they breakdown. This is termed their dielectric strength E_m.

4.18 The force between positive and negative charges is often described in terms of an electric flux linking them. This is measured using the same units as electric charge (coulombs) and thus a charge of Q coulombs will produce a total electric flux of Q coulombs. We also define what is termed the electric flux density, D, as the amount of flux passing through a defined area perpendicular to the flux.

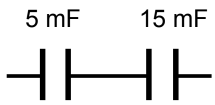
4.19 $D = Q/A = 35\ \mu C/(15\ mm \times 35\ mm) = 67\ mC/m^2$.

4.20 (a)



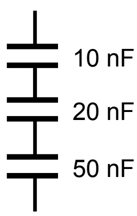
$$\begin{aligned} C &= C_1 + C_2 \\ &= 50\ \mu F + 100\ \mu F \\ &= 150\ \mu F \end{aligned}$$

(b)



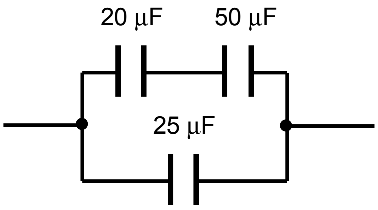
$$\begin{aligned} C &= 1(\ 1/C_1 + 1/C_2) \\ &= 1/(1/5\ mF + 1/15\ mF) \\ &= 3.75\ mF \end{aligned}$$

(c)



$$\begin{aligned} C &= 1(\ 1/C_1 + 1/C_2 + 1/C_3) \\ &= 1/(1/10\ nF + 1/20\ nF + 1/50\ nF) \\ &= 5.88\ nF \end{aligned}$$

(d)



$C = 25\ \mu F + 1/(1/20\ \mu F + 1/50\ \mu F)$
$= 25\ \mu F + 14.3\ \mu F$
$= 39.3\ \mu F$

4.21 In circuit (a) the 10 V supply is connected directly across C1 and C2, so the voltage across each of these devices is 10 V.

In circuit (b) the two capacitors are in series, so the charge on each capacitor must be the same. As the capacitance of two capacitors in series is equal the reciprocal of the sum of the reciprocals of the two capacitances, this combination has an effective capacitance C given by

$C = 1/(1/C_1 + 1/C_2)$

$= 1/(\ 1/10\ \mu F = 1/20\ \mu F)$

$= 6.667\ \mu F.$

Therefore, since the charge stored is equal to CV,

$Q = CV$

$= 6.667\ \mu F \times 10\ V$

$= 66.67\ \mu C.$

Therefore, the voltage across $C_3 = Q/C_3 = 66.67\ \mu C/10\ \mu F = 6.667\ V$, and the voltage across $C_4 = Q/C_4 = 66.67\ \mu C/20\ \mu F = 3.333\ V$.

4.22 In each case, the charge stored is equal to the product of the voltage and the capacitance. So

The charge stored in $C_1 = VC = 10\ V \times 10\ \mu F = 100\ \mu C$.

The charge stored in $C_2 = VC = 10\ V \times 20\ \mu F = 200\ \mu C$.

The charge stored in $C_3 = VC = 6.667\ V \times 10\ \mu F = 66.67\ \mu C$.

The charge stored in $C_4 = VC = 3.333\ V \times 20\ \mu F = 66.67\ \mu C$.

4.23

$$V = \frac{1}{C}\int I\mathrm{d}t$$

4.24 The time constant T of a circuit, is a constant, which determines the speed at which the circuit responds to change. For a circuit consisting of a resistor and a capacitor, the time constant is equal to the product *CR*. So in the circuit shown T = 10 μF × 50 kΩ = 0.5 s.

4.25 As the time constant is determined by the product of *C* and *R*, if *R* is *increased* by a factor of 10, then *C* must be *decreased* by a factor of 10 to leave the time constant unchanged. So in this case, *C* must be 1 μF.

4.26 The current waveform is also sinusoidal but is phase-shifted with respect to the voltage waveform by 90° (or π/2 radians) with the current waveform *leading* the voltage waveform.

4.27

$$E = \frac{1}{2}CV^2$$

4.28 $E = \frac{1}{2}CV^2 = \frac{1}{2} \times 0.005 \times 15^2 = 562$ mJ.

4.29 $E = \frac{1}{2}Q^2/C = \frac{1}{2} \times (1.25 \times 10^{-3})^2/50 \times 10^{-6} = 15.6$ mJ.