Answers to Exercises in Chapter 5

5.1 A wire carrying an electrical current causes a magnetomotive force (m.m.f.), F, which produces a magnetic field about it. One can think of an m.m.f. as being similar in some ways to an e.m.f. within an electric circuit.

5.2 This is as shown in Figure 5.1.

5.3 At a distance of 1 m from the wire, the circumference of the magnetic path is $l = 2\pi r = 6.28$ m. Therefore, $H = I/l = 3/6.28 = 0.48$ A/m. This is clockwise about the wire when looking in the direction of conventional current flow.

5.4 What factors determine the flux density at a particular point in space adjacent to a current carrying wire? The flux density at a point depends on the strength of the field at that point and the material present. Magnetic flux density is related to the field strength by the expression $B = \mu H$ where μ is the permeability of the material through which the field passes.

5.5 The permeability of free space μ_0 is the ratio of the magnetic flux density to the magnetic field strength in a vacuum. μ_0 has a value of $4\pi \times 10^{-7}$ henry/metre.

5.6 The relative permeability of a material μ_r is the ratio of the flux density produced in the material to that produced in a vacuum. As it is a ratio, it has no units. For a non-magnetic material, a typical value would be close to unity, while for a ferromagnetic material it might be 1,000 or more.

5.7 The magnetomotive force $F = IN$.

5.8 (i) The magnetomotive force is given by the 'ampere-turns' of the coil and is therefore given by

$$
\begin{aligned}
F &= IN \\
&= 5 \times 600 \\
&= 3000 \text{ ampere} - \text{turns}
\end{aligned}
$$

(ii) The magnetic field strength is given by the m.m.f. divided by the length of the magnetic path. In this case, the length of the magnetic path is the mean circumference of the coil, and so

$$
\begin{aligned}
H &= \frac{IN}{l} \\
&= \frac{3000}{0.9} \\
&= 3333 \text{ A/m}
\end{aligned}
$$

(iii) For a non-magnetic material $B = \mu_0 H$, and so

$$\begin{aligned} B &= \mu_0 H \\ &= 4\pi \times 10^{-7} \times 3333 \\ &= 4.19\,\text{mT} \end{aligned}$$

(iv)

$$\begin{aligned} \Phi &= BA \\ &= 4.2 \times 10^{-3} \times 400 \times 10^{-6} \\ &= 1.68\,\mu\text{Wb} \end{aligned}$$

5.9 If the toroid were replaced with a material with a μ_r of 500, this would have no effect on (i) and (ii), but would increase (iii) and (iv) by a factor of 500.

5.10 $S = F/\Phi = 15/(5 \times 10^{-3}) = 3{,}000$ A/Wb.

5.11 Faraday's law:

The magnitude of the e.m.f. induced in a circuit is proportional to the rate of change of the magnetic flux linking the circuit.

Lenz's law:

The direction of the e.m.f. is such that it tends to produce a current that opposes the change of flux responsible for inducing that e.m.f.

5.12 Inductance is the property whereby an e.m.f. is induced into a wire as a result of a changes in magnetic flux.

5.13 Self-inductance is the property whereby when the current within a coil changes, an e.m.f. is induced in that coil, which tends to oppose the change in the current.

5.14

$$V = L\frac{dI}{dt}$$

5.15 The henry can be defined as the inductance of a circuit when an e.m.f. of one volt is induced by a change in the current of one ampere per second.

5.16 $L = V/dI/dt = 150\ \mu V/50\ mA/s = 3\ mH$.

5.17 The presence of a ferromagnetic material dramatically increases the flux density within a coil and consequently also increases the rate of change of flux.

5.18

$$L = \frac{\mu_0 A N^2}{l}$$
$$= \frac{4\pi \times 10^{-7} \times 40 \times 10^{-6} \times 600^2}{500 \times 10^{-3}}$$
$$= 36\,\mu\text{H}$$

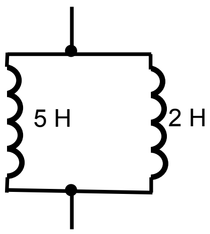
5.19

$$L = \frac{\mu_0 \mu_r A N^2}{l}$$
$$= \frac{4\pi \times 10^{-7} \times 800 \times 100 \times 10^{-6} \times 250^2}{300 \times 10^{-3}}$$
$$= 21\,\text{mH}$$

5.20 All real inductors have resistance. A real inductor can be modelled as an ideal inductor in series with a small resistance.

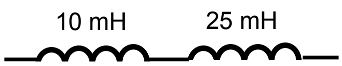
5.21 Even straight wires exhibit small amounts of inductance, therefore all connections within a circuit introduce inductance and the effects combine so that they can dramatically effect circuit behaviour.

5.22 (a)



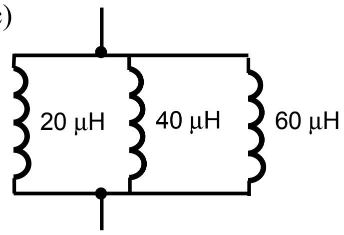
$L = 1/(1/L_1 + 1/L_2)$
$= 1/(1/5 + 1/2)$
$= 1.43$ H

(b)



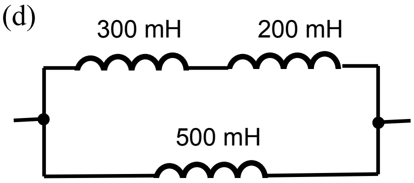
$L = L_1 + L_2$
$= 10$ mH + 25 mH = 35 mH

(c)



$L = 1/(1/L_1 + 1/L_2 + 1/L_3)$
$= 1/(1/(20\times10^{-6}) + 1/(40\times10^{-6}) + 1/(60\times10^{-6}))$
$= 10.9\ \mu$H

(d)



L = 1/(1/(300 mH + 200 mH)+1/500 mH)
= 1/(1/500 mH+1/500 mH)
= 250 mH

5.23

$$V = L\frac{\mathrm{d}I}{\mathrm{d}t}$$

5.24 The voltage produced by an inductor is related to the rate of change of the current within it. Therefore, a constant current will produce no voltage.

5.25 The current through an inductor cannot change instantaneously, as this would correspond to $\mathrm{d}I/\mathrm{d}t = \infty$, and would produce an infinite induced voltage opposing the change in current.

5.26 The time constant T of a circuit, is a constant, which determines the speed at which the circuit responds to change. For a circuit consisting of a resistor and an inductor, the time constant is equal to L/R.

So in the circuit shown T = 100 mH/50 Ω = 2 ms.

5.27 As the time constant is equal to L/R, if the resistance is increased by a factor of 10, then the inductance must also be increased by a factor of 10 to leave the time constant unchanged. Therefore, the required value of the inductor is 1 H.

5.28 When switching inductive loads very high induced voltages appear across the switch and cause 'arcing' at the switch contacts. This maintains the current for a short time after the switch is operated and reduces the rate of change of current. Arcing across switches can cause severe damage to the contacts and also generates electrical interference. For this reason, when it is necessary to switch inductive loads, we normally add circuitry to reduce the rate of change of the current.

5.29 The voltage across an inductor is given by $L\,\mathrm{d}I/\mathrm{d}t$, and so the voltage is directly proportional to the *time differential* of the current. As the differential of a sine wave is a cosine wave, the current waveform is phase-shifted with respect to the voltage waveform by 90°. The current waveform *lags* the voltage waveform.

5.30 $E = ½LI^2 = ½ \times 2 \times 10^{-3} \times 7^2 = 49$ mJ.

5.31 Mutual inductance is the property whereby if two conductors are linked magnetically, then a changing current in one of these will produce a changing magnetic flux associated with the other, and will result in an induced voltage in this second conductor.

5.32 The Henry is the mutual inductance between two circuits when an e.m.f. of one volt is induced in one, by a change in the current of one ampere per second in the other.

5.33 The coupling coefficient describes the degree of coupling between circuits, and is the fraction of the flux of one coil that links with the other.

5.34 The turns ratio of a transformer is the ratio of the number of turns in the secondary to that in the primary.

5.35 Under these circumstances, the voltage gain is equal to the turns ratio, therefore, we would expect the output voltage to be $10 \times 5 = 50$ V peak.

5.36 The addition of a resistor across the secondary coil would mean that current would flow in the seconday circuit. This current will produce a magnetic flux that will oppose that created by the primary coil. This will, therefore, reduce the output voltage. As the resistance is reduced, the output current will increase and the output voltage will fall.

5.37 A step-up transformer is one with more turns in the secondary than in the primary, so that the output voltage is greater than the input voltage, but it can deliver a smaller current.

5.38 A step-down transformer is one with less turns in the secondary than in the primary, so that the output voltage is less than the input voltage, but it can supply more current.

5.39 The dot notation is used to indicate the polarity of coil windings. Current flowing *into* each winding at the connection indicated by the dot will produce magnetomotive forces in the same direction within the core.

5.40 This is described in Section 5.14.

5.41 This is described in Section 5.14.