Answers to Exercises in Chapter 6

6.1 Angular frequency $\omega = 100$ rad/s; peak voltage = 15 V.

6.2 Angular frequency $\omega = 250$ rad/s, cyclic frequency $f = \omega/2\pi = 250/2\pi = 39.8$Hz. Its peak magnitude is 25 V, so its r.m.s. voltage is $25 \times 0.707 = 17.7$ V.

6.3 $v = 20 \sin 300t$.

6.4 $v = 20 \sin 314t$.

6.5 $v = 7 \sin (6.28 \times 10^3\, t)$.

6.6 $v = 6 \sin (6.28 \times 10^2\, t + 72°)$.

6.7 $v = i$ R.

6.8

$$v_L = L\frac{di}{dt}$$

6.9

$$v_C = \frac{1}{C}\int i dt$$

6.10 The voltage across a resistor is *in phase* with the current.

6.11 The voltage across a capacitor *lags* the current by 90°.

6.12 The voltage across an inductor *leads* the current by 90°.

6.13 The word CIVIL can be used as a mnemonic to represent the expression:

In C, I leads V; V leads I in L

as discussed in Section 6.2.

6.14 The reactance of a capacitor or inductance is the ratio of the voltage to the current, ignoring any phase shift, and is a measure of how the component opposes the flow of electricity.

6.15 The reactance of a resistor is zero.

6.16 The reactance of an inductor is ωL.

6.17 The reactance of a capacitor is $1/\omega C$.

6.18 $X_L = \omega L = 2\pi f L = 2 \times \pi \times 100 \times 20 \times 10^{-3} = 12.6\ \Omega$.

6.19 $X_C = 1/\omega C = 1/(500 \times 10 \times 10^{-9}) = 200\ \mathrm{k\Omega}$.

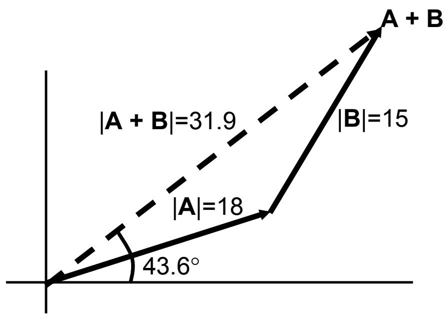
6.20 $X_C = 1/\omega C = 1/2\pi fC = 1/(2 \times \pi \times 250 \times 50 \times 10^{-6}) = 12.7\ \Omega$. $I = V/X_C = 15/12.7 = 1.18\mathrm{A}$.

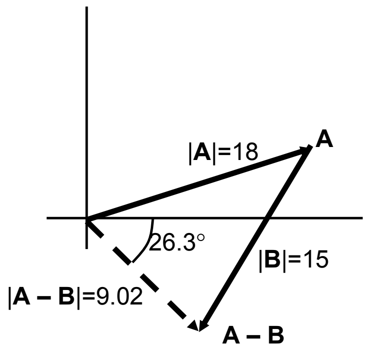
6.21 $X_L = \omega L = 100 \times 25 \times 10^{-3} = 2.5\ \Omega$. $V = I\,X_C = 2 \times 10^{-3} \times 2.5 = 5\ \mathrm{mV}$.

6.22 This is discussed in Section 6.4.

6.23 The length of the phasor represents the magnitude of the voltage, while the angle represents its phase angle with respect to some reference waveform.

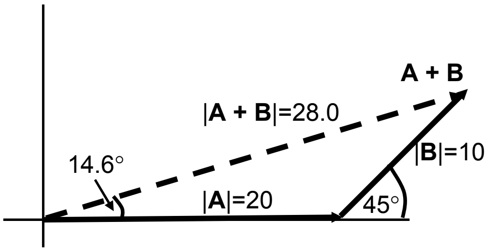
6.24





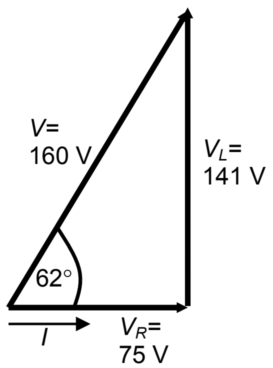
$|\mathbf{A} + \mathbf{B}| = 31.9$ $\angle(\mathbf{A} + \mathbf{B}) = 43.6°$
$|\mathbf{A} - \mathbf{B}| = 9.02$ $\angle(\mathbf{A} - \mathbf{B}) = -26.3°$

6.25



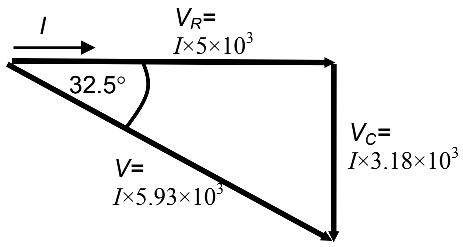
$|\mathbf{A} + \mathbf{B}| = 28.0$
$\angle(\mathbf{A} + \mathbf{B}) = 14.6°$

6.26 Voltage across resistor $V_R = IR = 3 \times 25 = 75$ V.
Reactance of inductor $X_L = 2\pi fL = 2 \times \pi \times 100 \times 75 \times 10^{-3} = 47.1\ \Omega$.
Therefore voltage across inductor $V_L = I\,X_L = 3 \times 47.1 = 141$ V.



$|\mathbf{V}| = 160$
$\angle\mathbf{V} = 62°$

6.27 Voltage across resistor $V_R = IR = I \times 5 \times 10^3$.
Reactance of capacitor $X_C = 1/\omega C = 1/2\pi fC = 1/(2 \times \pi \times 500 \times 100 \times 10^{-9}) =$ 3.18 kΩ. Therefore, voltage across capacitor $V_C = I\,X_C = I \times 3.18 \times 10^3$.



$|\mathbf{V}| = I \times 5.93 \times 10^3$
$\angle \mathbf{V} = -32.5°$

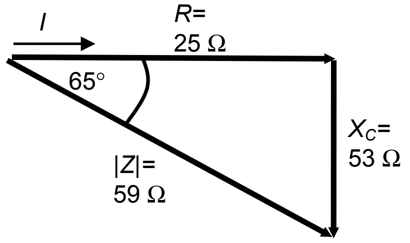
Since the voltage across the RC combination is known to be 12 V, it follows that

$$I \times 5.93 \times 10^3 = 12$$

$$I = \frac{12}{5.93 \times 10^3} = 2\,\text{mA}$$

Therefore, the magnitude of the current is 2 mA and voltage lags the current by 32.5° (or alternatively, the current leads the voltage by 32.5°).

6.28 The reactance of the capacitor $X_C = 1/2\pi fC = 1/(2 \times \pi \times 300 \times 10 \times 10^{-6}) = 53\ \Omega$. Therefore, the impedance is given by



$|\mathbf{Z}| = 59\ \Omega$
$\angle \mathbf{Z} = -65°$

6.29 If $x = 5 + j7$ and $y = 8 - j10$, then

$$(x + y) = 13 - j3$$

$$(x - y) = -3 + j17$$

$$(x \times y) = 110 + j6$$

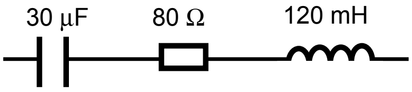
$$(x \div y) = -0.183 + j0.646$$

6.30 $\mathbf{Z_R} = 1\ \text{k}\Omega$.

6.31 $\mathbf{Z_C} = -jX_C = -j(1/2\pi fC) = -j(1/(2 \times \pi \times 10^3 \times 10^{-6})) = -j159\ \Omega$.

6.32 $\mathbf{Z_L} = jX_L = j(2\pi fL) = j(2 \times \pi \times 10^3 \times 10^{-3}) = j6.28\ \Omega$.

6.33 (a)



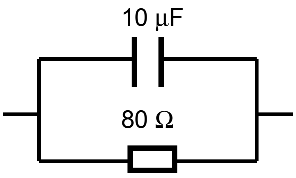
$\mathbf{Z_C} = -jX_C = -j(1/2\pi fC) = -j(1/(2 \times \pi \times 200 \times 30 \times 10^{-6})) = -j27\ \Omega$.

$\mathbf{Z_R} = 80\ \Omega$.

$\mathbf{Z_L} = jX_L = j(2\pi fL) = j(2 \times \pi \times 200 \times 120 \times 10^{-3}) = j151\ \Omega$.

Therefore, $\mathbf{Z} = \mathbf{Z_C} + \mathbf{Z_R} + \mathbf{Z_L} = -j27\ \Omega + 80\ \Omega + j151\ \Omega = 80 + j124\ \Omega$.

(b)



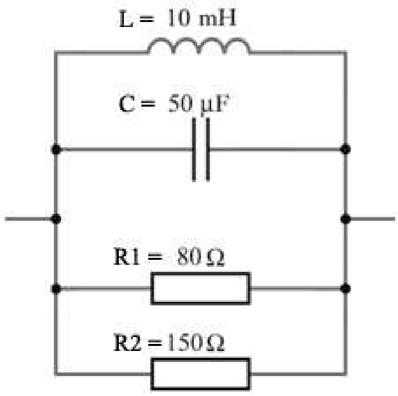
$\mathbf{Z_C} = -jX_C = -j(1/2\pi fC) = -j(1/(2 \times \pi \times 200 \times 10 \times 10^{-6})) = -j80\ \Omega$.

$\mathbf{Z_R} = 80\ \Omega$.

Therefore,

$$
\begin{aligned}
Z &= \frac{1}{\frac{1}{Z_C} + \frac{1}{Z_R}} \\
&= \frac{1}{\frac{1}{-j80} + \frac{1}{80}} \\
&= \frac{-j80 \times 80}{80 - j80} \\
&= \frac{-j80}{(1 - j)} \\
&= \frac{-j80}{(1 - j)} \frac{(1 + j)}{(1 + j)} \\
&= \frac{80 - j80}{2} \\
&= 40 - j40
\end{aligned}
$$

6.34



$\mathbf{Z_L} = jX_L = j(2\pi fL) = j(2 \times \pi \times 100 \times 10 \times 10^{-3}) = j6.28\ \Omega$.

$\mathbf{Z_C} = -jX_C = -j(1/2\pi fC) = -j(1/(2 \times \pi \times 100 \times 50 \times 10^{-6})) = -j31.8\ \Omega$.

$\mathbf{Z_{R1}} = 80\ \Omega$.

$\mathbf{Z_{R2}} = 150\ \Omega$.

Therefore,

$$
\begin{aligned}
Z &= \frac{1}{\frac{1}{Z_L} + \frac{1}{Z_C} + \frac{1}{Z_{R1}} + \frac{1}{Z_{R2}}} \\
&= \frac{1}{\frac{1}{j6.28} + \frac{1}{-j31.8} + \frac{1}{80} + \frac{1}{150}} \\
&= \frac{1}{-j\frac{1}{6.28} + j\frac{1}{31.8} + \frac{1}{80} + \frac{1}{150}} \\
&= \frac{1}{\left(\frac{1}{80} + \frac{1}{150}\right) + j\left(\frac{1}{31.8} - \frac{1}{6.28}\right)} \\
&= \frac{1}{0.0192 - j0.128} \\
&= \frac{(0.0192 + j0.128)}{(0.0192 - j0.128)(0.0192 + j0.128)} \\
&= \frac{(0.0192 + j0.128)}{(0.0192)^2 + (0.128)^2} \\
&= \frac{(0.0192 + j0.128)}{0.0167} \\
&= 1.15 + j7.66\Omega
\end{aligned}
$$

6.35 This can be achieved using a resistor of 1.15 Ω in series with an inductor with an impedance of 7.66 Ω.

Since $Z_L = jX_L = j(2\pi fL)$, it follows that $L = X_L/2\pi f$, so in this case

$$L = 7.66/2 \times \pi \times 100 = 12.2 \text{ mH}$$

Therefore, a suitable arrangement might look like this.

6.36 Using the same circuit as in Exercise 6.34, but at a frequency of 200 Hz:

$\mathbf{Z_L} = jX_L = j(2\pi fL) = j(2 \times \pi \times 200 \times 10 \times 10^{-3}) = j12.6\ \Omega$.

$\mathbf{Z_C} = -jX_C = -j(1/2\pi fC) = -j(1/(2 \times \pi \times 200 \times 50 \times 10^{-6})) = -j15.9\ \Omega$.

$\mathbf{Z_{R1}} = 80\ \Omega$.

$\mathbf{Z_{R2}} = 150\ \Omega$.

Therefore,

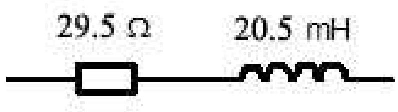
$$\begin{aligned}
Z &= \frac{1}{\frac{1}{Z_L} + \frac{1}{Z_C} + \frac{1}{Z_{R1}} + \frac{1}{Z_{R2}}} \\
&= \frac{1}{\frac{1}{j12.6} + \frac{1}{-j15.9} + \frac{1}{80} + \frac{1}{150}} \\
&= \frac{1}{-j\frac{1}{12.6} + j\frac{1}{15.9} + \frac{1}{80} + \frac{1}{150}} \\
&= \frac{1}{\left(\frac{1}{80} + \frac{1}{150}\right) + j\left(\frac{1}{15.9} - \frac{1}{12.6}\right)} \\
&= \frac{1}{0.0192 - j0.0167} \\
&= \frac{(0.0192 + j0.0167)}{(0.0192 - j0.0167)(0.0192 + j0.0167)} \\
&= \frac{(0.0192 + j0.0167)}{(0.0192)^2 + (0.0167)^2} \\
&= \frac{(0.0192 + j0.0167)}{0.000649} \\
&= 29.5 + j25.8\Omega
\end{aligned}$$

6.37 This can be achieved using a resistor of 29.5 Ω in series with an inductor with an impedance of 25.8 Ω.

Since $Z_L = jX_L = j(2\pi fL)$, it follows that $L = X_L/2\pi f$, so in this case

$$L = 25.8/2 \times \pi \times 200 = 20.5 \text{ mH}$$

Therefore a suitable arrangement might look like this.



6.38 $36\angle 56°$, $36\ e^{j56}$.

6.39 $19.1 - j16.1$, $25\ e^{-j40}$.

6.40 The impedance of the inductance $\mathbf{Z_L} = jX_L = j\omega L = j314 \times 50 \times 10^{-3} = j15.7\ \Omega$. Therefore the impedance of the entire circuit $\mathbf{Z} = \mathbf{Z_R} + \mathbf{Z_L} = 10 + j15.7\ \Omega$. Converting to polar coordinates this gives $18.6\angle 57.5°$

If the input voltage is taken as the reference phase the input is $60\angle 0°$.

The resulting current I is given by

$$\begin{aligned} I &= \frac{V}{Z} \\ &= \frac{60\angle 0°}{18.6\angle 57.5°} \\ &= 3.22\angle -57.5° \end{aligned}$$

Thus, the magnitude of the current is 3.22 A and the current lags the applied voltage by 57.7°.

6.41 Impedance of the capacitance $\mathbf{Z_C} = -jX_C = -j(1/\omega C) = -j(1/(377 \times 5 \times 10^{-6})) = -j530\ \Omega$. Therefore, the impedance of the entire circuit $\mathbf{Z} = 1/(1/\mathbf{Z_R} + 1/\mathbf{Z_C})$.

$$\begin{aligned}
Z &= \frac{1}{\frac{1}{Z_C}+\frac{1}{Z_R}} \\
&= \frac{1}{\frac{1}{-j530}+\frac{1}{1{,}000}} \\
&= \frac{-j530\times 1{,}000}{1{,}000 - j530} \\
&= \frac{-j530{,}000(1{,}000 + j530)}{(1{,}000 - j530)(1{,}000 + j530)} \\
&= \frac{(2.8\times 10^{8}) + j(5.3\times 10^{8})}{1.28\times 10^{6}} \\
&= 219 - j414
\end{aligned}$$

Converting to polar coordinates this gives $468\angle{-62°}$

If the input current is taken as the reference phase the input is $0.5\angle 0°$.

The resulting voltage V is given by

$$\begin{aligned}
V &= IZ \\
&= 0.5\angle 0° \times 468\angle -62° \\
&= 234\angle -62°
\end{aligned}$$

Thus, the magnitude of the voltage is 234 V and the voltage lags the input current by 62°.