Answers to Exercises in Chapter 7

7.1 As discussed in Section 7.2, the instantaneous power varies at twice the frequency of the applied voltage. So in this case it varies at 100 Hz.

7.2 The power dissipated in the resistor is VI were V is the r.m.s. voltage and I is the r.m.s. current. $VI = V^2/R = (V_p/\sqrt{2})^2/R = V_p^{\,2}/2R = 10^2/(2 \times 50) = 1$ W.

7.3 As discussed in Section 7.3, the instantaneous power varies at twice the frequency of the applied voltage. So in this case it varies at 100 Hz.

7.4 As discussed in Section 7.3, the average power dissipation is zero.

7.5 As discussed in Section 7.4 the instantaneous power varies at twice the frequency of the applied voltage. So in this case it varies at 100 Hz.

7.6 As discussed in Section 7.3, the average power dissipation is zero.

7.7 These terms are discussed in Section 7.5.

7.8

$$\begin{aligned}\text{Apparent power } S &= VI\\ &= 100\times 7\\ &= 700 \text{ VA}\end{aligned}$$

$$\begin{aligned}\text{Power factor} &= \cos\phi\\ &= \cos\ 60^\circ\\ &= 0.5\end{aligned}$$

$$\begin{aligned}\text{Active power } P &= S\cos\phi\\ &= 700\times 0.5\\ &= 350 \text{ W}\end{aligned}$$

7.9 This is discussed in Section 7.6.

7.10 If f = 50 Hz then $\omega = 2\pi f = 314$ rad/s. Therefore, the impedance of the arrangement $\mathbf{Z} = R + j\omega L = 40 + j(314 \times 0.1) = 40 + j31.4\ \Omega$. This can be converted to polar coordinates to give $\mathbf{Z} = 50.8\angle 38.1°$.

The current i is given by $v/\mathbf{Z} = 100\angle 0°/50.8\angle 38.1° = 1.97\ \angle{-38.1}°$. Therefore, the r.m.s. current is 1.97 A. The apparent power is $VI = 100 \times 1.97 = 197$ var. The power factor is $\cos\ \phi = \cos -38.1° = 0.787$. The active power = $VI \cos\ \phi = 100 \times 1.97 \times 0.787 = 155$ W. The reactive power = $VI \sin\ \phi = 100 \times 1.97 \times 0.617 = 121$ W.

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7.11 The apparent power S of the motor is 500 VA, since this is the rating of the motor. The power factor ($\cos\phi$) is 0.8. Therefore, the active power in the motor is

$$\begin{aligned}\text{Active Power } P &= S\cos\phi \\ &= 500\times 0.8 \\ &= 400 \text{ watts}\end{aligned}$$

Since $\cos\phi = 0.8$, it follows that $\sin\phi = \sqrt{1-\cos^2\phi} = 0.6$. Therefore,

$$\begin{aligned}\text{Reactive power } Q &= S\sin\phi \\ &= 500\times 0.6 \\ &= 300 \text{ var}\end{aligned}$$

The current is given by the apparent power divided by the voltage

$$\begin{aligned}\text{Current } I &= \frac{S}{V} \\ &= \frac{500}{250} \\ &= 2 \text{ A}\end{aligned}$$

7.12 This is discussed in Section 7.7.

7.13 In Exercise 7.11, we determined that for this motor:

$$\begin{aligned}\text{Apparent power } S &= 500 \text{ VA} \\ \text{Active power } P &= 400 \text{ watts} \\ \text{Current } I &= 2 \text{ A} \\ \text{Reactive power } Q &= 300 \text{ var}\end{aligned}$$

The capacitor is required to cancel the *lagging* reactive power. We therefore need to add a capacitive element with a *leading* reactive power Q_C of −300 var.

Now, just as $P = V^2/R$, so $Q = V^2/X$. Since capacitive reactive power is negative,

$$Q_C = -\frac{250^2}{X_C} = -300 \text{ var}$$

$$X_C = \frac{250^2}{300} = 208.3\,\Omega$$

$X_C = 1/\omega C$ which is equal to $1/2\pi fC$. Therefore,

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$$\frac{1}{2\pi fC} = 208.3$$

$$C = \frac{1}{208.3 \times 2 \times \pi \times f}$$

$$= \frac{1}{208.3 \times 2 \times 3.142 \times 60}$$

$$= 12.7\ \mu\text{F}$$

7.14 The addition of power factor correction does not affect the active power, which is therefore 400 W. If the power factor is 0.9, this implies that the apparent power is 400/0.9 = 444.4 VA. Since the power factor is equal to cos ϕ, it follows that $\phi = \cos^{-1} 0.9 = 25.84°$, and that the reactive power = 444.4 sin ϕ = 193.7 var. Therefore, in order to reduce the power factor to 0.9, we need to add a capacitor to bring the reactive power down from 300 to 193.7. This requires a capacitor to supply a reactive power of 193.7 − 300 = −106.3 var.

Therefore,

$$Q_C = -\frac{250^2}{X_C} = -106.3 \text{ var}$$

$$X_C = \frac{250^2}{106.3} = 588\ \Omega$$

$X_C = 1/\omega C$, which is equal to $1/2\pi fC$. Therefore,

$$\frac{1}{2\pi fC} = 588$$

$$C = \frac{1}{588 \times 2 \times \pi \times f}$$

$$= \frac{1}{588 \times 2 \times 3.142 \times 60}$$

$$= 4.5\ \mu\text{F}$$

7.15 At 50 Hz, the reactance of the inductor $X_L = \omega L = 2\pi fL = 2 \times 3.142 \times 50 \times 16 \times 10^{-3} = 5.03\ \Omega$. Therefore, the impedance of the load is 10 + j5.03 Ω. This can be converted to a polar form as $11.2\angle 26.9°\ \Omega$. The applied voltage has a magnitude of 20 V peak which is $20/\sqrt{2}$ = 14.14 V. Therefore, $V = 14.14\angle 0°$. The current $I = V/\mathbf{Z} = V = 14.14\angle 0°/11.2\angle 26.9° = 1.26\angle{-26.9°}$. Thus, the r.m.s. current is 1.26 A. Therefore, $\phi = -26.9°$ and the power factor = cos ϕ = 0.892. The apparent power = VI = 14.14 × 1.26 = 17.8 VA, and the active power = 17.8 × 0.892 = 15.9 watts.

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7.16 The required capacitor can be calculated by determining the reactive power in the circuit and calculating the value of capacitance required to compensate for this (as in Exercise 7.15 above). Alternatively, we can note that in order to produce a power factor of 1.0 the capacitive reactance must equal the inductive reactance, and therefore $1/\omega C = \omega L$, which can be solved to give $C = 633\ \mu F$.

When the reactive power is zero, the applied voltage appears across the resistor so the peak current will be 20/10 = 2A. Therefore, the active power is $VI = 20/\sqrt{2} \times 2/\sqrt{2} = 20$ W.

7.17 Where three conductors are used each provides one of the phases, and loads are connected between the conductors. In a four-line system, the additional wire is a neutral conductor. Loads may then be connected between each phase and neutral.

7.18 When using sinusoidal signals the power dissipated within a load is determined not only by the r.m.s. values of the voltage and current, but also by the phase angle between the voltage and current waveforms, (which determines the power factor).

7.19 In single-phase AC circuits, power is normally measured using an electrodynamic wattmeter. This device passes the load current through a series of low-resistance field coils, and places the load voltage across a high-resistance armature coil. The resulting deflection is directly related to the product of the instantaneous current and voltage, and hence to the instantaneous power in the load.

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