Answers to Exercises in Chapter 8

8.1 A two-port network is a circuit configuration that has two 'ports' – these are an input port and an output port.

8.2 As discussed in Section 8.2:

Voltage gain (A_v) = V_o/V_i

Current gain (A_i) = I_o/I_i

Power gain (A_p) = P_o/P_i

8.3

Voltage gain (A_v) = V_o/V_i = 15 V/2 V = 7.5

Current gain (A_i) = I_o/I_i = 10 mA/1 mA = 10

Power gain (A_p) = P_o/P_i = ($V_o \times I_o$)/($V_i \times I_i$)
= (15 V × 10 mA)/(2 V × 1 mA)
= (150 mW)/(2 mW)
= 75

8.4 The overal gain is the product of the gain of each stage. Therefore,

Power gain = 75 × 25 × 0.3 = 562.5

8.5 The gain of each stage in decibels is give by

1st stage – power gain = 75, so gain in dB = 10 $\log_{10}$ 75 = 18.75 dB

2nd stage – power gain = 25, so gain in dB = 10 $\log_{10}$ 25 = 13.98 dB

3rd stage – power gain = 0.5, so gain in dB = 10 $\log_{10}$ 0.3 = −5.229 dB

So the gain of the three stages in series is 18.75 + 13.98 − 5.229 = 27.501 dB

This corresponds to a power gain of 562.5 as determined in Example 8.5.

8.6 If the gain of a ciruit is 25 dB, the gain as a ratio is $10^{25/10}$ = 316.2.

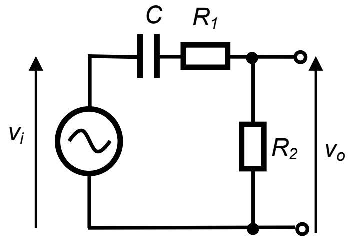
8.7 If the gain of a circuit is 25 dB, the voltage gain is $10^{25/20}$ = 17.8.

8.8 $X_C = 1/\omega C = 1/2\pi fC = 1/(2 \times \pi \times 10^4 \times 10^{-6}) = 15.9\ \Omega$.
$X_L = \omega L = 100 \times 20 \times 10^{-3} = 2\ \Omega$.

8.9 $f = \omega/2\pi = 250/2\pi = 39.8$ Hz.

8.10 $\omega = 2\pi f = 2 \times \pi \times 250 = 1{,}571$ rad/s.

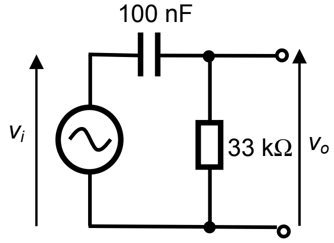
8.11



$$\frac{v_o}{v_i} = \frac{\mathbf{Z_2}}{\mathbf{Z_1} + \mathbf{Z_2}}$$
$$= \frac{R_2}{R_2 + R_1 - \mathrm{j}\dfrac{1}{\omega C}}$$

8.12 T = $CR = 15 \times 10^{-9} \times 33 \times 10^{3} = 495$ µs.

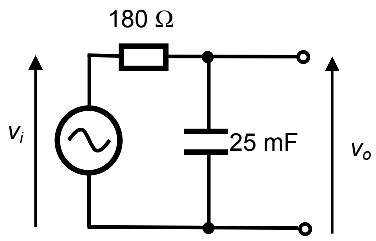
8.13



T = $CR = 100 \times 10^{-9} \times 33 \times 10^{3} = 3.3$ ms. $\omega_c = 1/\mathrm{T} = 1/3.3$ ms $= 303$ rad/s. $f_c = \omega_c/2\pi = 303/2\pi = 48.2$ Hz. This is a low-frequency cut-off.

8.14 An octave below 30 Hz is 15 Hz;
two octaves above 25 kHz is 100 kHz;
three octaves above 1 kHz is 8 kHz;
a decade above 1 MHz is 10 MHz;
two decades below 300 Hz is 3 Hz;
three decades above 50 Hz is 50 kHz.

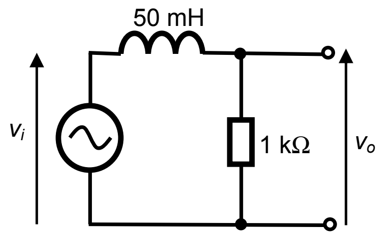
8.15



T = $CR = 25 \times 10^{-3} \times 180 = 4.5$ s. $\omega_c = 1/\mathrm{T} = 1/4.5 = 0.222$ rad/s. $f_c = \omega_c/2\pi = 0.222/2\pi = 35.4$ mHz. This is a high-frequency cut-off.

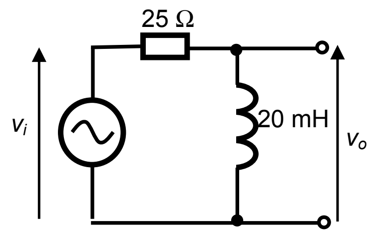
8.16 T = $L/R = 30 \times 10^{-3}/150 = 200$ µs.

8.17



T = L/R = 50 × 10^{-3}/1 × 10^3 = 50 µs. ω_c = 1/T = 1/(50 × 10^{-6}) = 20 krad/s. f_c = $\omega_c/2\pi$ = (20 × 10^3)/2π = 3.18 kHz. This is a high-frequency cut-off.

8.18



T = L/R = 20 × 10^{-3}/25 = 800 µs. ω_c = 1/T = 1/(800 × 10^{-6}) = 1,250 rad/s. f_c = $\omega_c/2\pi$ = (20 × 10^3)/2π = 199 Hz. This is a low-frequency cut-off.

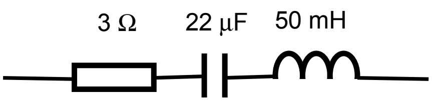
8.19 The straight-line approximations should resemble those in Figure 8.11(a), with the cut-off frequency f_c = 199 Hz. The more realistic curves should resemble those in Figure 8.12(a), again with f_c = 199 Hz.

8.20 At high frequencies, the gain will fall at 18 dB per octave increase in frequency. At low frequencies, it will fall at 12 dB per octave decrease in frequency.

8.21 At high frequencies the gain will tend to −270°, at low frequencies it will tend to +180°.

8.22 Resonance is discussed in Section 8.9.

8.23



$$
\begin{aligned}
f_0 &= \frac{\mathbf{1}}{2\pi\sqrt{LC}} \\
&= \frac{1}{2\pi\sqrt{(50\times10^{-3}\times22\times10^{-6})}} \\
&= 152 \text{ Hz}
\end{aligned}
$$

$$
\begin{aligned}
Q &= \frac{1}{R}\sqrt{\frac{L}{C}} \\
&= \frac{1}{3}\sqrt{\frac{50\times10^{-3}}{22\times10^{-6}}} \\
&= 15.9
\end{aligned}
$$

$$
\begin{aligned}
B &= \frac{R}{2\pi L} \\
&= \frac{3}{2\times\pi\times50\times10^{-3}} \\
&= 9.55\text{Hz}
\end{aligned}
$$

8.24

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{50\text{ mH } \times 22\mu\text{F}}} = 152\text{ Hz}$$

$$Q = R\sqrt{\frac{C}{L}} = 1\text{ k}\Omega\sqrt{\frac{22\ \mu\text{F}}{50\text{ mH}}} = 21$$

$$B = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 1\text{ k}\Omega \times 22\mu\text{F}} = 7.2\text{ Hz}$$

8.25 RC circuits are more commonly used than RL circuits, because inductors are expensive, bulky and have greater losses than capacitors.

8.26 Passive filters contain no active components and examples include *RC* and *RL* filters. Active filters incorporate an active amplifying element, such as an op-amp.

8.27 Inductors are often avoided in the construction of filters because they are expensive, bulky and suffer from greater losses than other passive components.

8.28 The Butterworth filter is optimised to produce a flat response within its pass-band.

8.29 The Chebyshev filter is optimised to produce a sharp transition from the pass-band to the stop-band.

8.30 The Bessel filter is optimised for a linear phase response and is sometimes called a linear phase filter.

8.31 The effects of stray capacitance and inductance are discussed in Section 8.11.