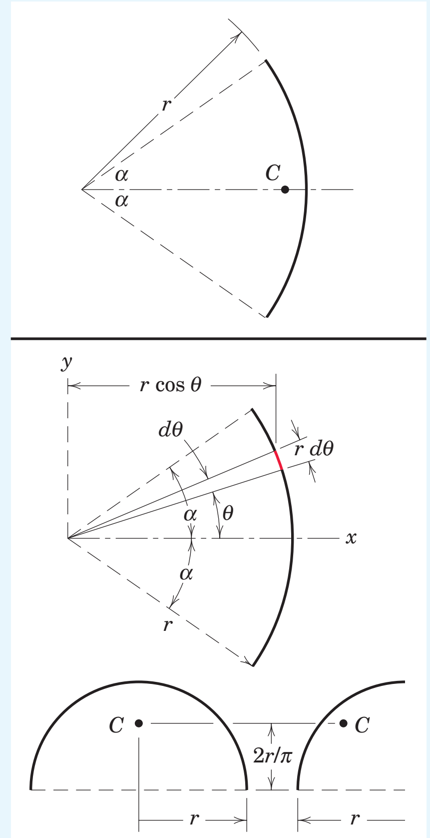
SAMPLE PROBLEM 5/1

Centroid of a circular arc. Locate the centroid of a circular arc as shown in the figure.



Solution. Choosing the axis of symmetry as the x-axis makes $\bar{y} = 0$. A differential element of arc has the length $dL = r\,d\theta$ expressed in polar coordinates, ① and the x-coordinate of the element is $r \cos \theta$.

Applying the first of Eqs. 5/4 and substituting $L = 2\alpha r$ give

$$[L\bar{x} = \int x\,dL] \qquad (2\alpha r)\bar{x} = \int_{-\alpha}^{\alpha} (r \cos \theta)\, r\, d\theta$$

$$2\alpha r\bar{x} = 2r^2 \sin \alpha$$

$$\bar{x} = \frac{r \sin \alpha}{\alpha} \qquad \textit{Ans.}$$

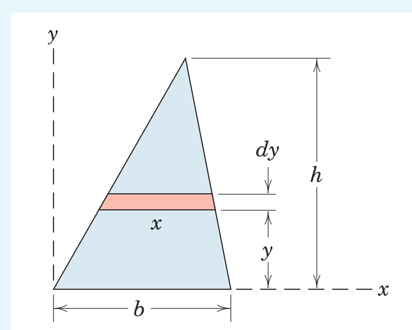
For a semicircular arc $2\alpha = \pi$, which gives $\bar{x} = 2r/\pi$. By symmetry we see immediately that this result also applies to the quarter-circular arc when the measurement is made as shown.

Helpful Hint

① It should be perfectly evident that polar coordinates are preferable to rectangular coordinates to express the length of a circular arc.

SAMPLE PROBLEM 5/2

Centroid of a triangular area. Determine the distance $\bar{h}$ from the base of a triangle of altitude h to the centroid of its area.



Solution. The x-axis is taken to coincide with the base. A differential strip of ① area $dA = x\,dy$ is chosen. By similar triangles $x/(h - y) = b/h$. Applying the second of Eqs. 5/5a gives

$$[A\bar{y} = \int y_c\,dA] \qquad \frac{bh}{2}\bar{y} = \int_0^h y\frac{b(h-y)}{h}\,dy = \frac{bh^2}{6}$$

and

$$\bar{y} = \frac{h}{3} \qquad \textit{Ans.}$$

This same result holds with respect to either of the other two sides of the triangle considered a new base with corresponding new altitude. Thus, the centroid lies at the intersection of the medians, since the distance of this point from any side is one-third the altitude of the triangle with that side considered the base.

Helpful Hint

① We save one integration here by using the first-order element of area. Recognize that dA must be expressed in terms of the integration variable y; hence, $x = f(y)$ is required.