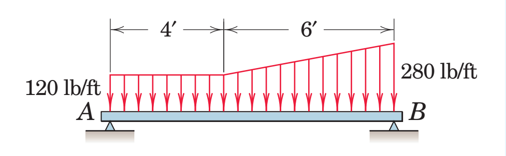
## SAMPLE PROBLEM 5/11

Determine the equivalent concentrated load(s) and external reactions for the simply supported beam which is subjected to the distributed load shown.




***Solution.*** The area associated with the load distribution is divided into the rectangular and triangular areas shown. The concentrated-load values are determined by computing the areas, and these loads are located at the centroids of the respective areas. ①

Once the concentrated loads are determined, they are placed on the free-body diagram of the beam along with the external reactions at $A$ and $B$. Using principles of equilibrium, we have

$[\Sigma M_A = 0]$ $\qquad 1200(5) + 480(8) - R_B(10) = 0$

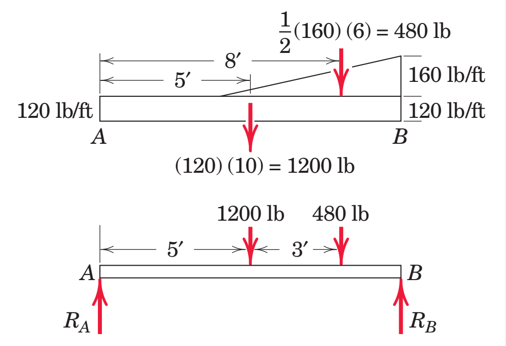
$R_B = 984$ lb *Ans.*

$[\Sigma M_B = 0]$ $\qquad R_A(10) - 1200(5) - 480(2) = 0$

$R_A = 696$ lb *Ans.*

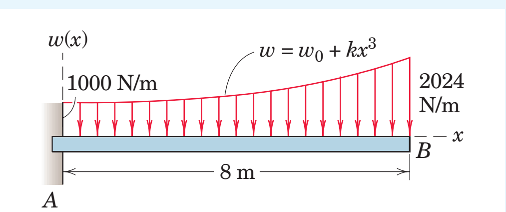
### Helpful Hint

① Note that it is usually unnecessary to reduce a given distributed load to a *single* concentrated load.




## SAMPLE PROBLEM 5/12

Determine the reaction at the support $A$ of the loaded cantilever beam.




***Solution.*** The constants in the load distribution are found to be $w_0 = 1000$ N/m and $k = 2$ N/m$^4$. The load $R$ is then ①

$$R = \int w\,dx = \int_0^8 (1000 + 2x^3)\,dx = \left(1000x + \frac{x^4}{2}\right)\Bigg|_0^8 = 10\,050 \text{ N}$$

② The $x$-coordinate of the centroid of the area is found by

$$\bar{x} = \frac{\int xw\,dx}{R} = \frac{1}{10\,050}\int_0^8 x(1000 + 2x^3)\,dx$$

$$= \frac{1}{10\,050}(500x^2 + \tfrac{2}{5}x^5)\Big|_0^8 = 4.49 \text{ m}$$

From the free-body diagram of the beam, we have

$[\Sigma M_A = 0]$ $\qquad M_A - (10\,050)(4.49) = 0$

$M_A = 45\,100$ N·m *Ans.*

$[\Sigma F_y = 0]$ $\qquad A_y = 10\,050$ N *Ans.*

Note that $A_x = 0$ by inspection.

### Helpful Hints

① Use caution with the units of the constants $w_0$ and $k$.

② The student should recognize that the calculation of $R$ and its location $\bar{x}$ is simply an application of centroids as treated in Art. 5/3.