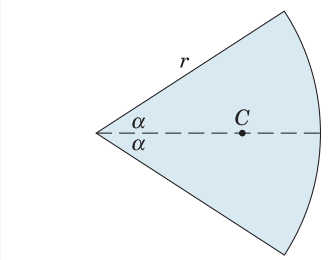
SAMPLE PROBLEM 5/3

Centroid of the area of a circular sector. Locate the centroid of the area of a circular sector with respect to its vertex.



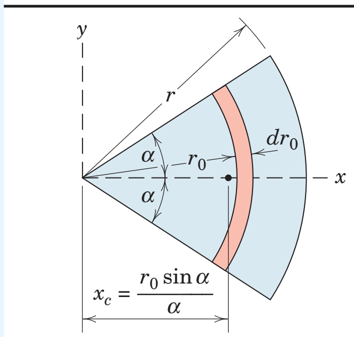
Solution I. The x-axis is chosen as the axis of symmetry, and $\bar{y}$ is therefore automatically zero. We may cover the area by moving an element in the form of a partial circular ring, as shown in the figure, from the center to the outer periphery. The radius of the ring is r_0 and its thickness is dr_0, so that its area is ① $dA = 2r_0\alpha\, dr_0$.

The x-coordinate to the centroid of the element from Sample Problem 5/1 is ② $x_c = r_0 \sin\alpha/\alpha$, where r_0 replaces r in the formula. Thus, the first of Eqs. 5/5a gives

$$[A\bar{x} = \int x_c\, dA] \qquad \frac{2\alpha}{2\pi}(\pi r^2)\bar{x} = \int_0^r \left(\frac{r_0 \sin\alpha}{\alpha}\right)(2r_0\alpha\, dr_0)$$

$$r^2\alpha\bar{x} = \tfrac{2}{3}r^3 \sin\alpha$$

$$\bar{x} = \frac{2}{3}\frac{r\sin\alpha}{\alpha} \qquad \textit{Ans.}$$



Solution I

Helpful Hints

① Note carefully that we must distinguish between the variable r_0 and the constant r.

② Be careful not to use r_0 as the centroidal coordinate for the element.

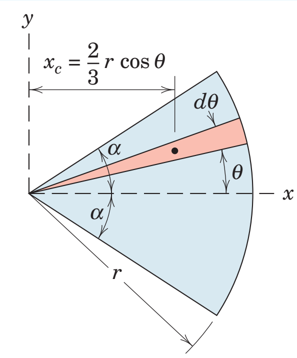
Solution II. The area may also be covered by swinging a triangle of differential area about the vertex and through the total angle of the sector. This triangle, shown in the illustration, has an area $dA = (r/2)(r\, d\theta)$, where higher-order terms are neglected. From Sample Problem 5/2 the centroid of the triangular element of area is two-thirds of its altitude from its vertex, so that the x-coordinate to the centroid of the element is $x_c = \frac{2}{3}r\cos\theta$. Applying the first of Eqs. 5/5a gives

$$[A\bar{x} = \int x_c\, dA] \qquad (r^2\alpha)\bar{x} = \int_{-\alpha}^{\alpha} (\tfrac{2}{3}r\cos\theta)(\tfrac{1}{2}r^2\, d\theta)$$

$$r^2\alpha\bar{x} = \tfrac{2}{3}r^3\sin\alpha$$

and as before

$$\bar{x} = \frac{2}{3}\frac{r\sin\alpha}{\alpha} \qquad \textit{Ans.}$$



Solution II

For a semicircular area $2\alpha = \pi$, which gives $\bar{x} = 4r/3\pi$. By symmetry we see immediately that this result also applies to the quarter-circular area where the measurement is made as shown.

It should be noted that, if we had chosen a second-order element $r_0\, dr_0\, d\theta$, one integration with respect to θ would yield the ring with which *Solution I* began. On the other hand, integration with respect to r_0 initially would give the triangular element with which *Solution II* began.