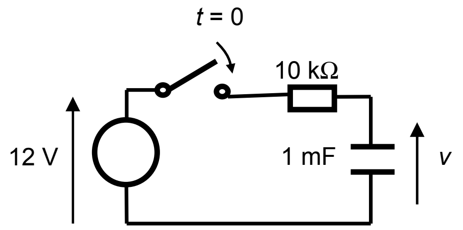
Answers to Exercises in Chapter 9

9.1 The 'steady state response' of a circuit is the behaviour of the circuit in response to either fixed DC signals or constant AC signals. The 'transient response' describes the behaviour of the circuit before it reaches this steady state condition. For example, how the circuit reacts when a voltage or current source is initially turned on or off.

9.2 Initially, there is no voltage across the capacitor so the applied voltage appears across the resistor and $i = V/R$. The steady state current in the circuit is zero, since a capacitor cannot pass a direct current.

9.3



The time constant of the circuit T = $CR = 10 \times 10^3 \times 1 \times 10^{-3} = 10$ s.

$$v = V\left(1 - e^{-\frac{t}{T}}\right)$$
$$= 12\left(1 - e^{-\frac{t}{10}}\right)$$

At $t = 4$ s

$$v = 12\left(1 - e^{-\frac{4}{10}}\right)$$
$$= 3.96 \text{ V}$$

9.4 The time constant T = $CR = 100 \times 10^{-6} \times 47 \times 10^3 = 4.7$ s.

Therefore, from Equation 9.3 we have

$$v = V\left(1 - e^{-\frac{t}{T}}\right)$$
$$= 15\left(1 - e^{-\frac{t}{4.7}}\right)$$

At $t = 5$ s,

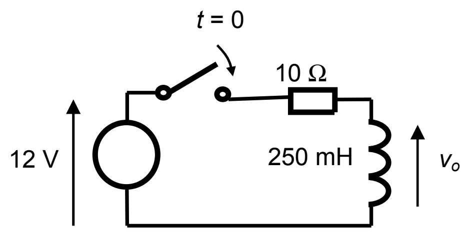
$$v = 15\left(1 - e^{-\frac{5}{4.7}}\right)$$
$$= 9.82 \text{ V}$$

9.5 The voltage on the capacitor will increase until it is equal to the applied voltage. Therefore, the final value of this voltage is 15 V.

9.6 When the capacitor is fully charged no current will flow in the circuit, so the voltage across the resistor will be 0 V.

9.7 The current in the inductor cannot change instantly; therefore, the initial current is zero. In the steady state, the current is equal to *V/R*.

9.8



The time constant of the circuit T = *L/R* = 0.25 ÷ 10 = 25 ms.

Therefore,

$$v = V\mathrm{e}^{-\frac{t}{\mathrm{T}}}$$

The time taken for *v* to fall to 12 V is given by

$$8 = 12\mathrm{e}^{-\frac{t}{0.025}}$$

$$0.667 = \mathrm{e}^{-\frac{t}{0.025}}$$

$$\ln(0.667) = -\frac{t}{0.025}$$

$$-0.405 = -\frac{t}{0.025}$$

$$t = 10 \text{ ms}$$

9.9 The time constant T = *L/R* = 50 × 10^{-3}/4.7 = 10.6 ms. The final current is given by *V/R* = 15 V/4.7 Ω = 3.19 A.

Therefore, from Equation 9.8 we have

$$i = I\left(1 - e^{-\frac{t}{T}}\right)$$
$$= 3.19\left(1 - e^{-\frac{t}{10.6\times10^{-3}}}\right)$$

At t = 20 ms,

$$i = 3.19\left(1 - e^{-\frac{20\times10^{-3}}{10.6\times10^{-3}}}\right) = 2.7 \text{ A}$$

9.10 In the steady state, the current through the inductor is constant, so the voltage across the inductor is zero.

9.11 In the steady state, since the voltage across the inductor is zero, the voltage across the resistor is equal to the applied voltage, which is 15 V.

9.12 From Equation 9.9 of the text

$$v = Ve^{-\frac{t}{T}}$$

Here V = 50 V and the time constant T = CR = 25 × 10^{-6} × 1 × 10^{3} = 25 ms. Therefore

$$v = 50e^{-\frac{t}{0.025}}$$

This drops to 10 V when

$$\begin{aligned} 10 &= 50e^{-\frac{t}{0.025}} \\ 0.2 &= e^{-\frac{t}{0.025}} \\ \ln 0.2 &= -\frac{t}{0.025} \\ -1.61 &= -\frac{t}{0.025} \\ t &= 40 \text{ ms} \end{aligned}$$

9.13 From Equation 9.11 of the text

$$i = Ie^{-\frac{t}{T}}$$

Here I = 1 A and the time constant T = L/R = 25 × 10^{-3}/100 = 250 µs. Therefore

$$i = 1e^{-\frac{t}{25\times10^{5}}}$$

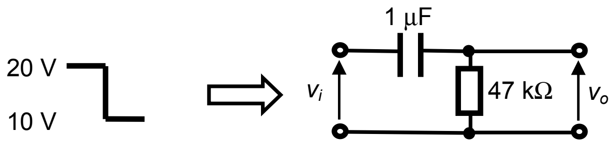
This reaches 100 mA when

$$\begin{aligned} 0.1 &= 1e^{-\frac{t}{25\times10^{-5}}} \\ \ln 0.1 &= -\frac{t}{25\times10^{-5}} \\ -2.30 &= -\frac{t}{25\times10^{-5}} \\ t &= 576\ \mu\text{s} \end{aligned}$$

9.14 'First-order systems' are those that can be described by first-order differential equations. Circuits containing resistance and either capacitance *or* inductance fall within this category.

9.15 The initial and final value formulae are described in Section 9.4.

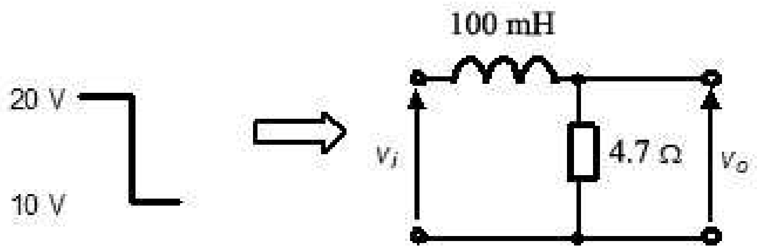
9.16



The time constant of the circuit T = $CR = 1 \times 10^{-6} \times 47 \times 10^3 = 47$ ms.
Before the step change, the capacitor will have charged to 20 V and the output voltage will be 0 V. When the step occurs, the capacitor voltage cannot change instantly, so the change in voltage (−10 V) will appear across the resistance. Therefore, the initial output voltage $V_i = -10$ V and the final value $V_f = 0$ V. Therefore,

$$\begin{aligned} v &= V_f + (V_i - V_f)\mathrm{e}^{-t/\mathrm{T}} \\ &= 0 + (-10 - 0)\mathrm{e}^{-t/0.047} \\ &= -10\mathrm{e}^{-t/0.047} \end{aligned}$$

9.17



The time constant of the circuit T = $L/R = 100 \times 10^{-3}/4.7 = 21.3$ ms. Before the step change the inductor will be de-energised. There will therefore be zero volts across the inductor and the output voltage will be 20 V. When the step occurs the current cannot change instantly, so the voltage across the resistor will initially be unchanged, but over time this will decrease to 10 V as the current stabilises. Therefore, $V_i = 20$ V and $V_f = 10$ V.

Therefore,

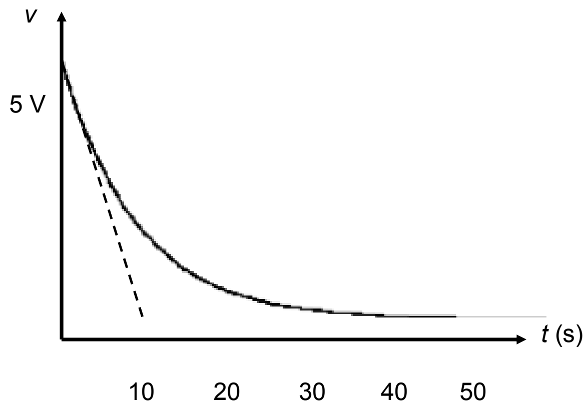
$$v = V_f + (V_i - V_f)e^{-\frac{t}{T}}$$

$$= 10 + (20 - 10)e^{-\frac{t}{0.0213}}$$

9.18 A saturating exponential waveform, is one that changes exponentially towards its final value – as shown in Figure 9.5(a).

9.19 A decaying exponential waveform is one that changes exponentially towards its final value – as shown in Figure 9.5(b).

9.20



9.21 Appropriate waveforms for each of these inputs, for each of the circuits, are given in Figure 9.7.

9.22 This occurs when the period of the input waveform is long compared with the time constant of the filter network.

9.23 This occurs when the period of the input waveform is short compared with the time constant of the filter network.

9.24 Arrangements that are described by second-order differential equations are termed 'second-order systems'. Circuits that contain both capacitance and inductance are normally second-order systems but some other circuit configurations also fall within this group.

9.25 Applying Kirchhoff's voltage law to this circuit gives

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri + v_C = V$$

Now, v_c is given by

$$v_C = \frac{1}{C}\int i\,\mathrm{d}t$$

Therefore, substituting in the earlier equation gives

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri + \frac{1}{C}\int i\,\mathrm{d}t = V$$

and differentiating with respect to t gives

$$L\frac{\mathrm{d}^2 i}{\mathrm{d}t^2} + R\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{1}{C}i = 0$$

The current in the circuit is given by the solution of this equation, which again is a second-order differential equation.

9.26 The 'undamped natural frequency' and 'damping factor' are explained and discussed in Section 9.5.

9.27 This corresponds to the condition that produces the fastest response in the absence of overshoot. Critical damping occurs when the damping factor ζ is equal to 1.