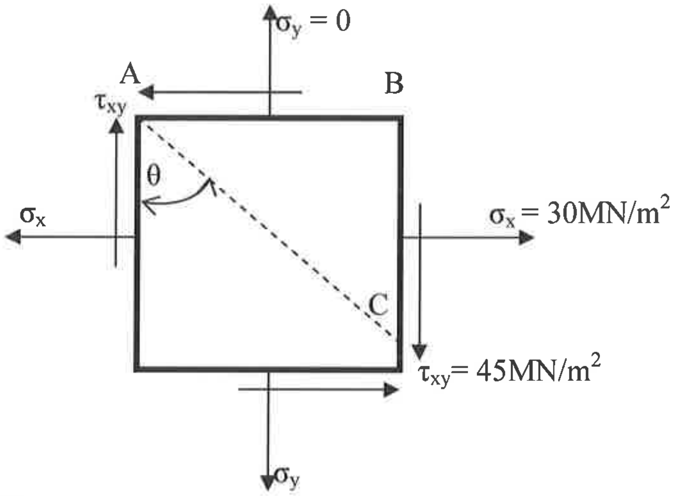
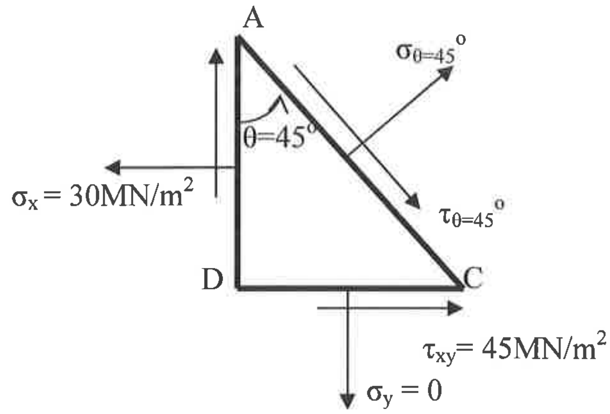
Worked Example – Analytical Solution
2D Stress Analysis

A machine component is subjected to direct loading, bending and torsion such that the 2D stress state is as shown in the figure.

(a) Calculate the principal stresses acting in the component.
(b) Determine the maximum shear stress.
(c) Determine the angle of the principal plane.
(d) Evaluate the stresses acting on the 45° plane shown in the figure.





Solution

(a)

From element: $\sigma_x = 30MN/m^2$ (tension), $\sigma_y = 0$, $\tau_{xy} = -45MN/m^2$ (CW)

Max. principal stress: $$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \frac{30+0}{2} + \sqrt{\left(\frac{30-0}{2}\right)^2 + 45^2} = 62.43MN/m^2$$ (tension)

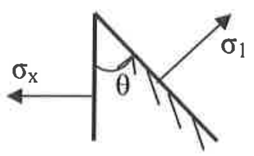
Min. principal stress: $$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \frac{30+0}{2} - \sqrt{\left(\frac{30-0}{2}\right)^2 + 45^2} = -32.43MN/m^2$$ (compression)

(b)

Max. shear stress: $$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \frac{\sigma_1 - \sigma_2}{2} = \frac{62.43-(-32.43)}{2} = 47.43MN/m^2$$

(c)

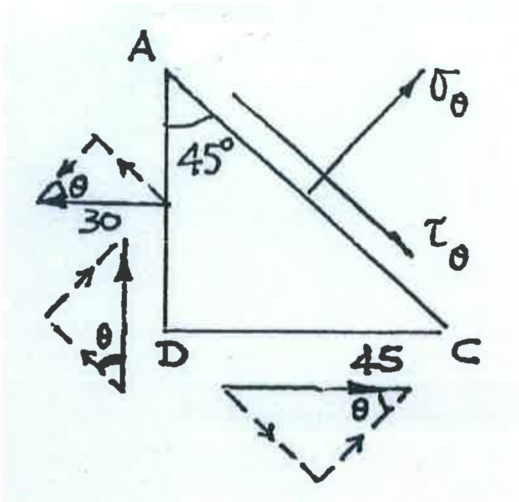
Angle of max. principal stress σ_1:



$$\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2x45}{30-0} = 3 \qquad \therefore \qquad \theta = 35.8^o$$

(d)

Stresses on 45° plane:



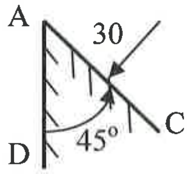
σ_θ and τ_θ are +ve as shown. Equating all parallel forces to $\theta = 45°$, in σ_θ direction where AD/AC = cos45° and DC/AC = sin45°:

σ_θ.AC.t = 30.AD.t.cos45° - 45.AD.t.sin45° - 45.DC.t.cos45

$$\therefore \quad \sigma_\theta = 30\frac{AD}{AC}\cos45° - 45\frac{AD}{AC}\sin45° - 45\frac{DC}{AC}\cos45°$$

$$= 30\text{x}0.7071^2 - 45\text{x}0.7071^2 - 45\text{x}0.7071^2$$

$$\therefore \quad \sigma_\theta = \underline{-30\ \text{MN/m}^2\ \text{(compressive)}}$$



Now, τ_θ.AC.t = 30.AD.t.sin45° + 45.AD.t.cos45° - 45.DC.t.sin45°

$$\therefore \quad \tau_\theta = 30\frac{AD}{AC}.\sin45° + 45\frac{AD}{AC}.\cos45° - 45\frac{DC}{AC}.\sin45°$$

$$= 30\sin45°\cos45° + 45\cos^2 45° - 45\sin^2 45°$$

$$= 15 + 22.5 - 22.5$$

$$\therefore \quad \tau_\theta = \underline{15\ \text{MN/m}^2\ \text{(i.e. +ve as assumed)}}$$