Glasgow Caledonian University

University for the Common Good

School of Engineering & Built Environment

African Leadership College, Mauritius

Programme:

MEng/BEng(Hons)
in
Electrical Power Systems Engineering

Glasgow Caledonian University

University for the Common Good

School of Engineering & Built Environment

Module:

Engineering Design & Analysis 2 (M2H721926)

2D Stress Analysis: Worked Example

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2D Stress Analysis
Worked Example

A component is subjected to two mutually perpendicular direct stresses of **80 MN/m^2 (tensile)** and **50 MN/m^2 (compressive)**, together with an ACW **shear stress of 30 MN/m^2**. Sketch an element of the component material showing the stresses acting, and, by drawing a ***Mohr Circle of Stress***, determine:

i) the magnitude and nature of the principal stresses;

ii) the magnitude of the maximum shear stress;

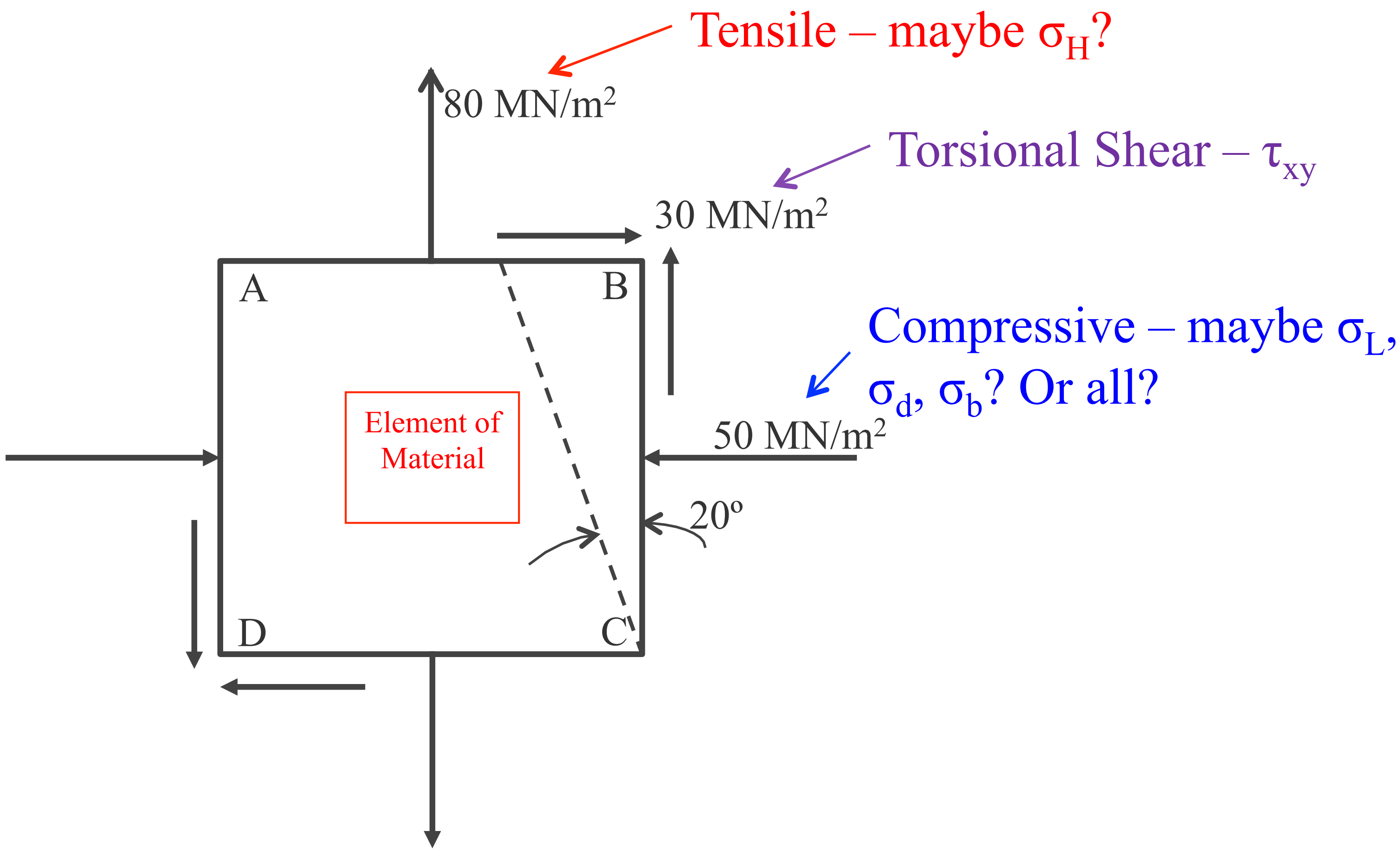
iii) the magnitude of the normal stress on a plane inclined at 20° counter-clockwise to the plane on which the 50 MN/m^2 stress acts.

iv) the magnitude of the shear stress on a plane inclined at 20° counter-clockwise to the plane on which the 50 MN/m^2 stress acts.

Check out the analytical solution!

Solution:

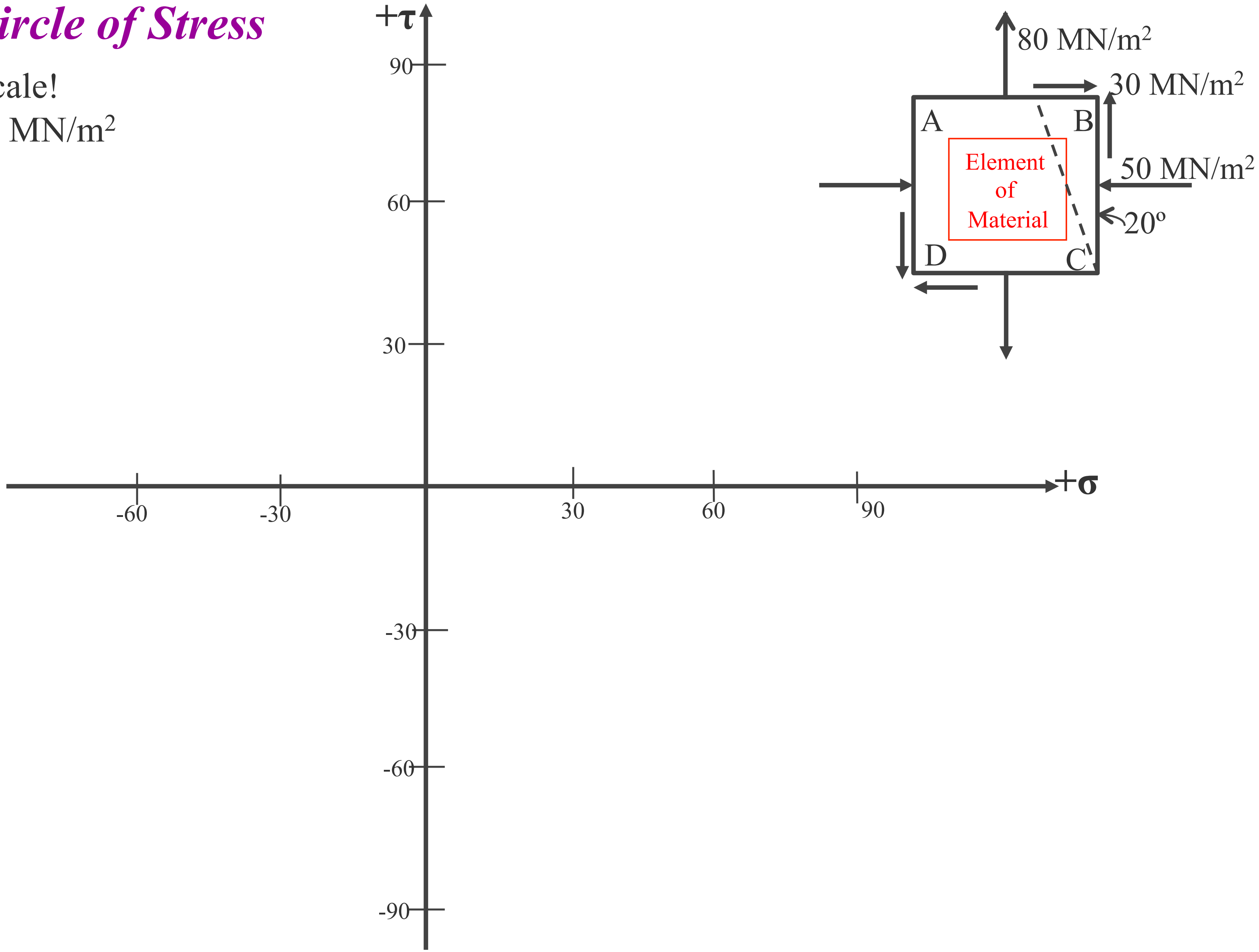
Element of material showing 2D stress state:



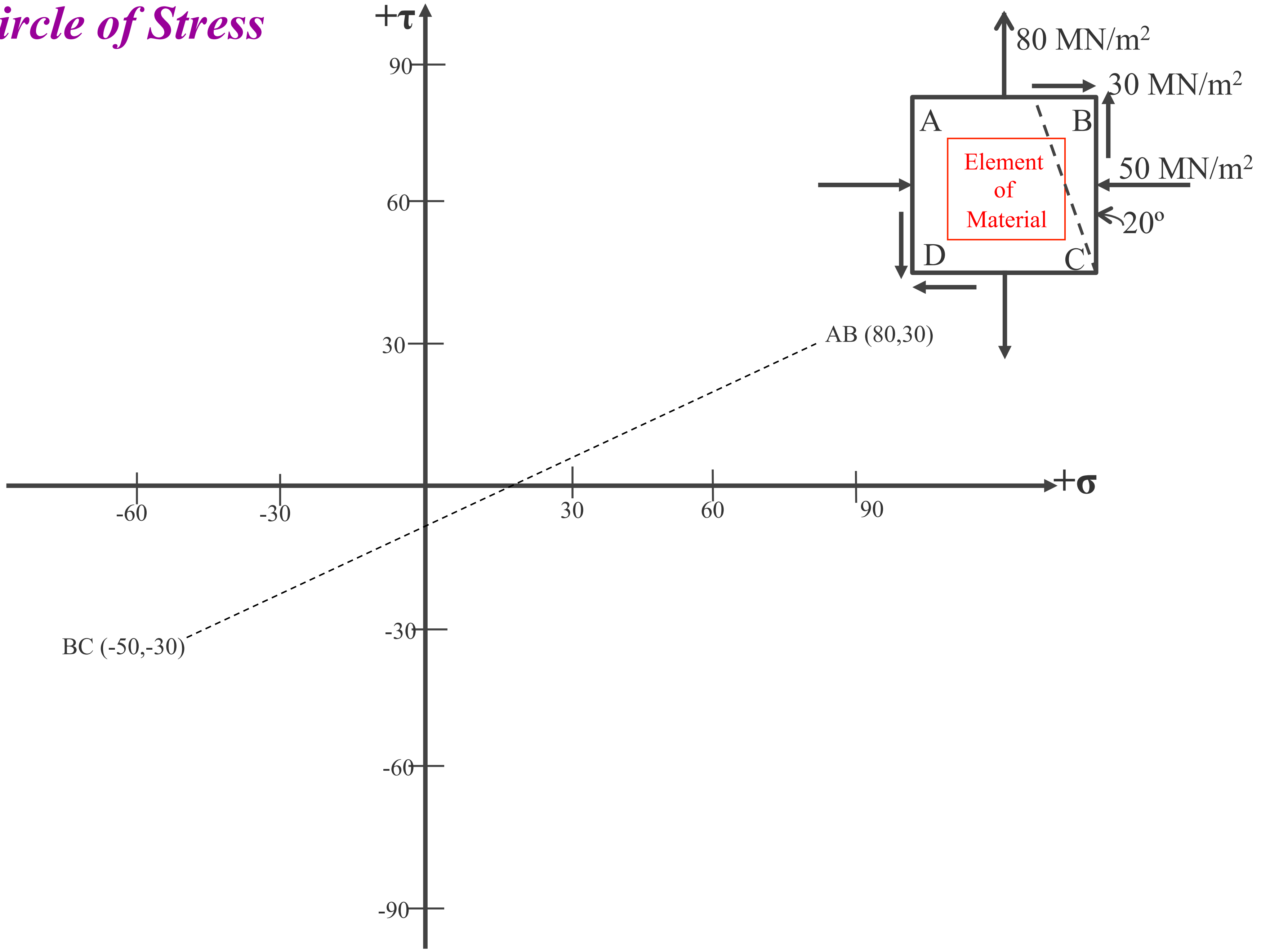
Mohr Circle of Stress

Choose Scale!

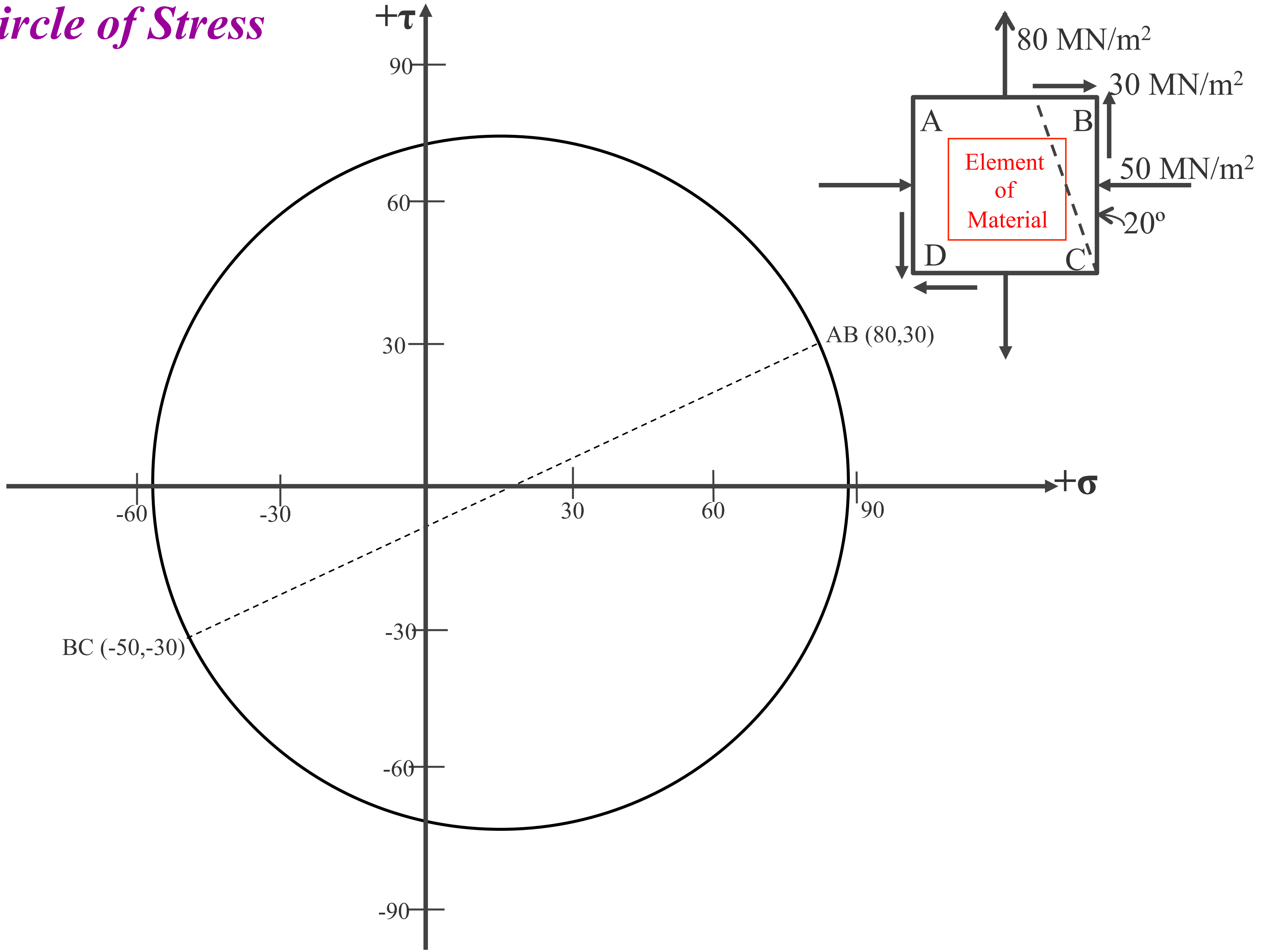
1mm : 1.2 MN/m²



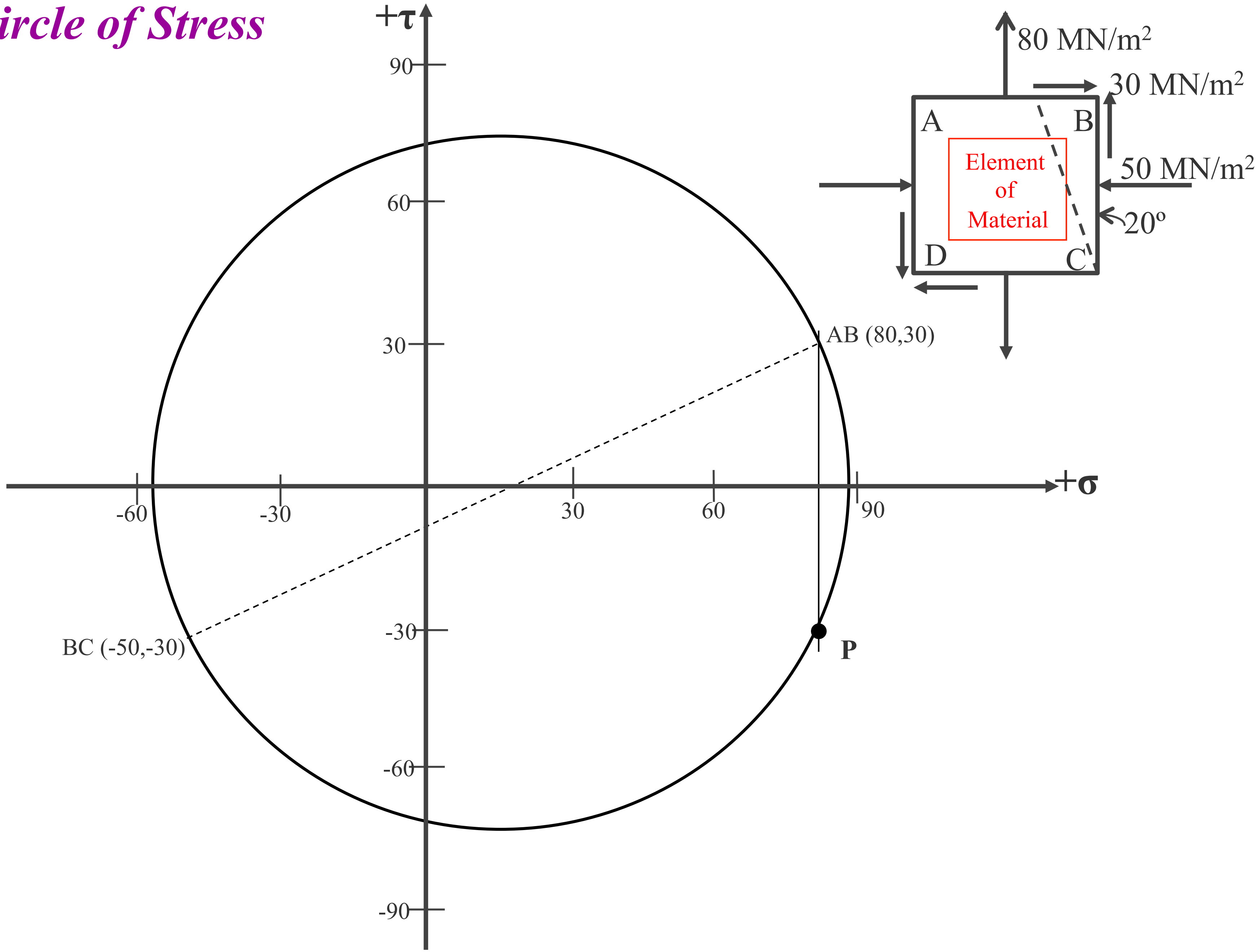
Mohr Circle of Stress



Mohr Circle of Stress



Mohr Circle of Stress

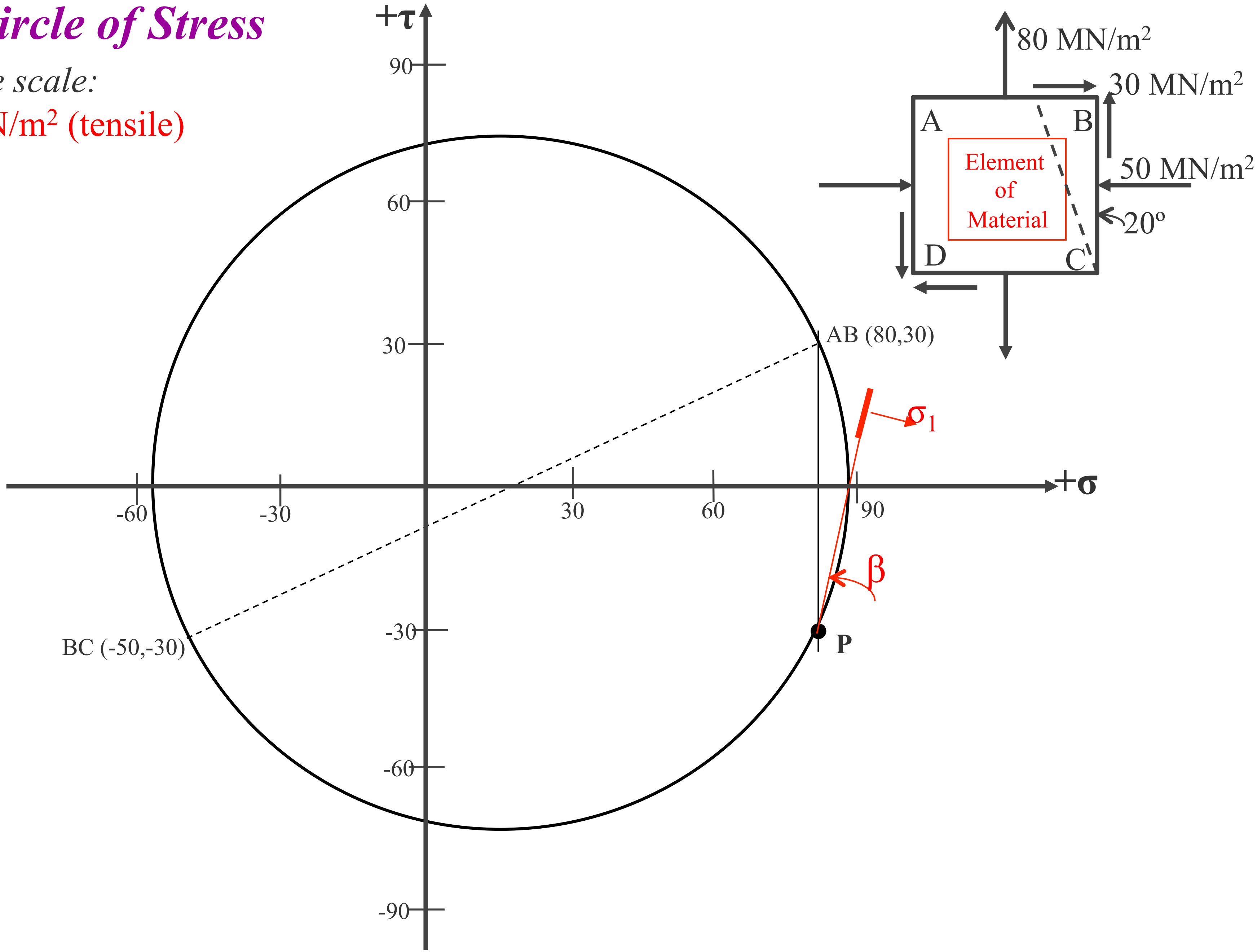


Mohr Circle of Stress

From circle scale:

σ_1 = 87 MN/m^2 (tensile)

β = 13°



Mohr Circle of Stress

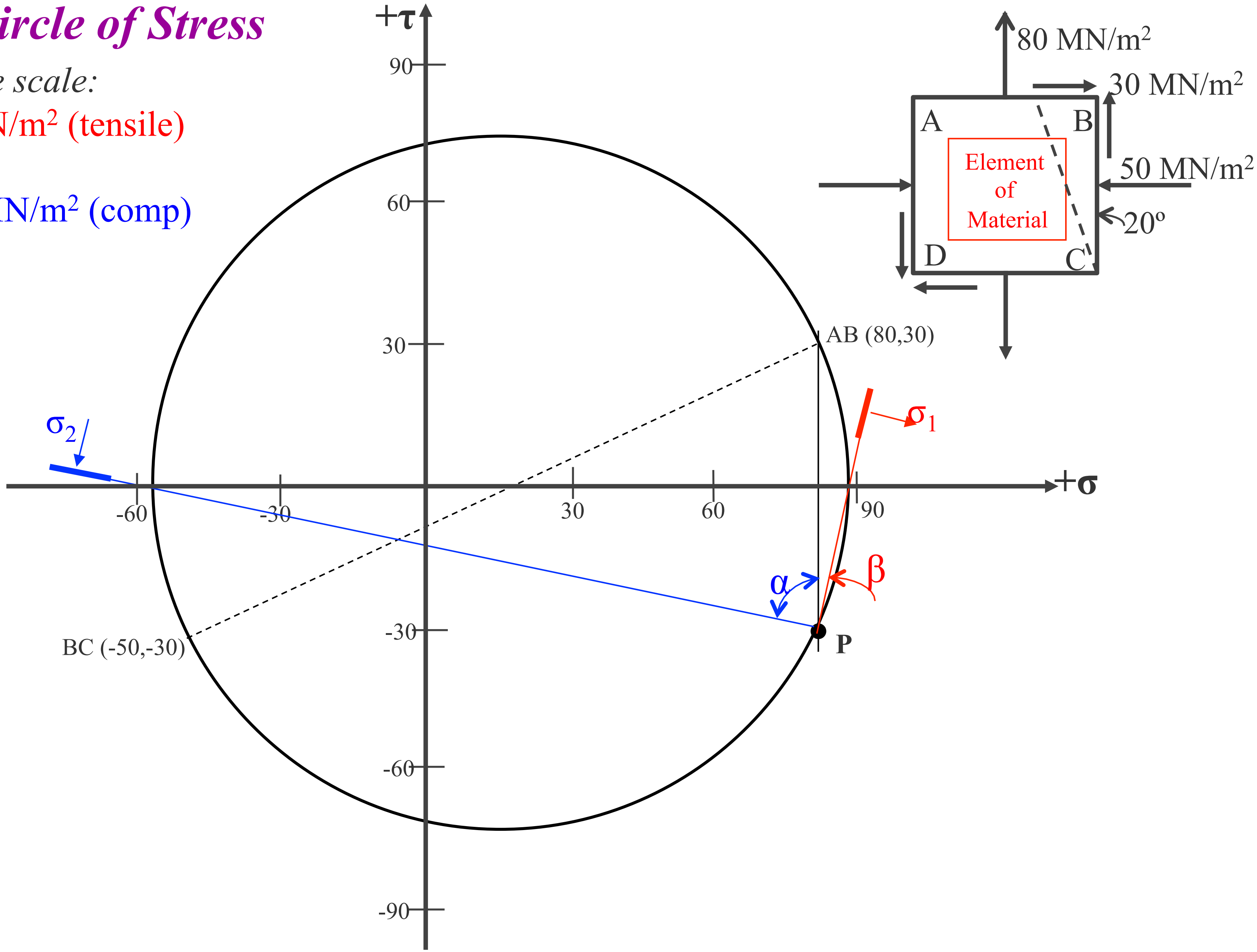
From circle scale:

σ_1 = 87 MN/m^2 (tensile)

β = 13°

σ_2 = - 57 MN/m^2 (comp)

α = 77°



Mohr Circle of Stress

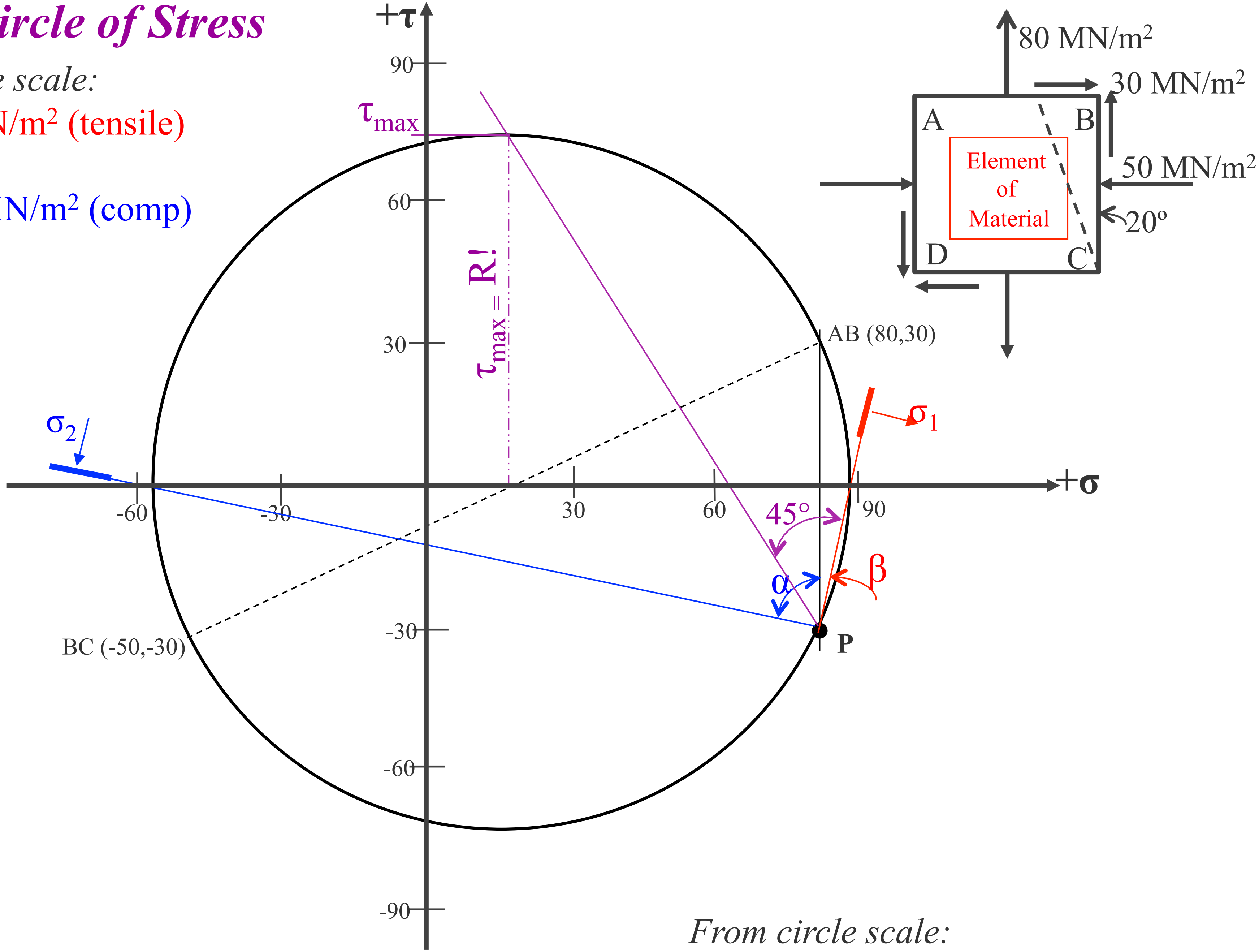
From circle scale:

σ_1 = 87 MN/m^2 (tensile)

β = 13°

σ_2 = - 57 MN/m^2 (comp)

α = 77°



From circle scale:

τ_{max} = 72 MN/m^2 (at 45° to σ_1 and σ_2)

Mohr Circle of Stress

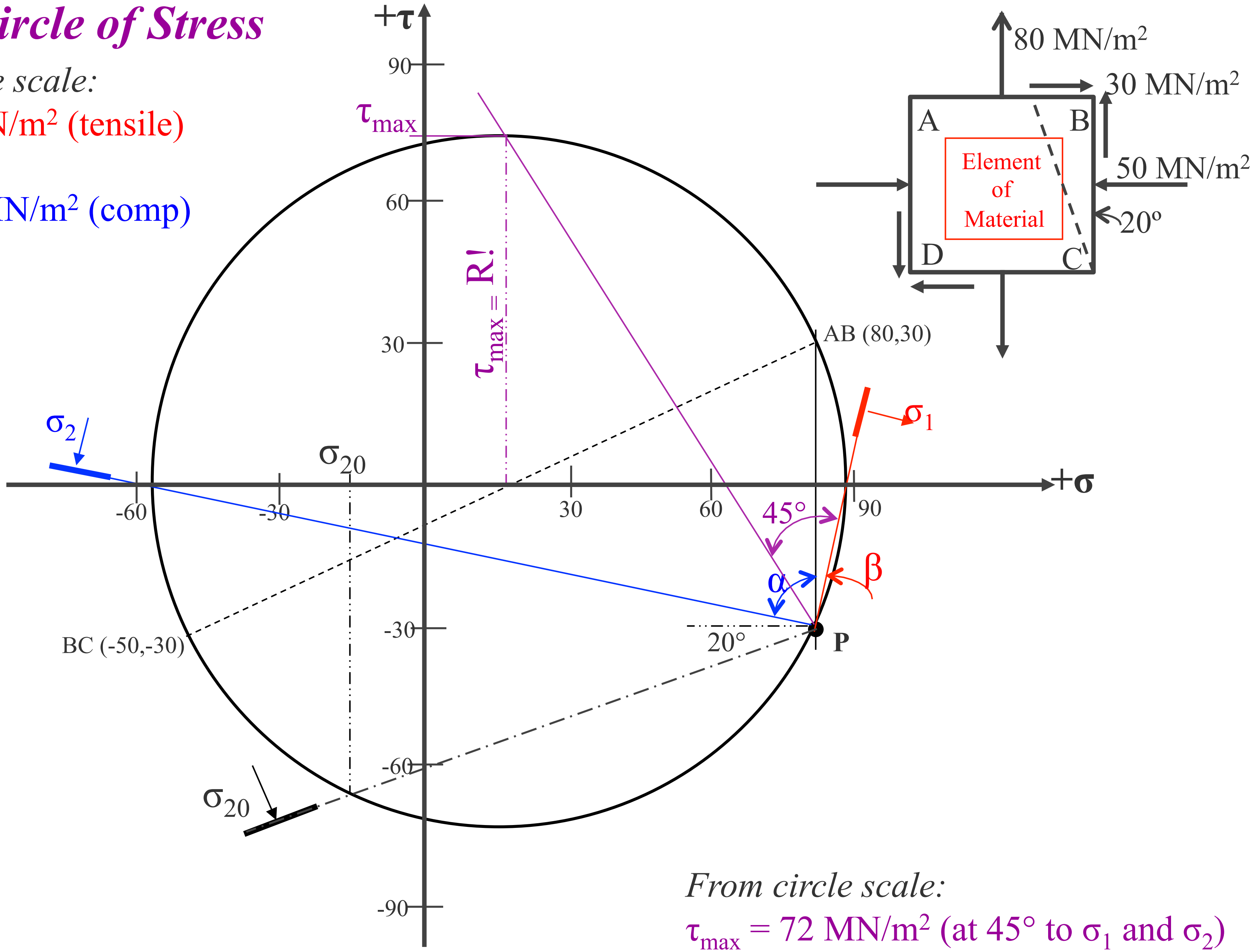
From circle scale:

σ_1 = 87 MN/m² (tensile)

β = 13°

σ_2 = - 57 MN/m² (comp)

α = 77°



From circle scale:

τ_{max} = 72 MN/m² (at 45° to σ_1 and σ_2)

σ_{20} = - 16 MN/m² (comp.)

Mohr Circle of Stress

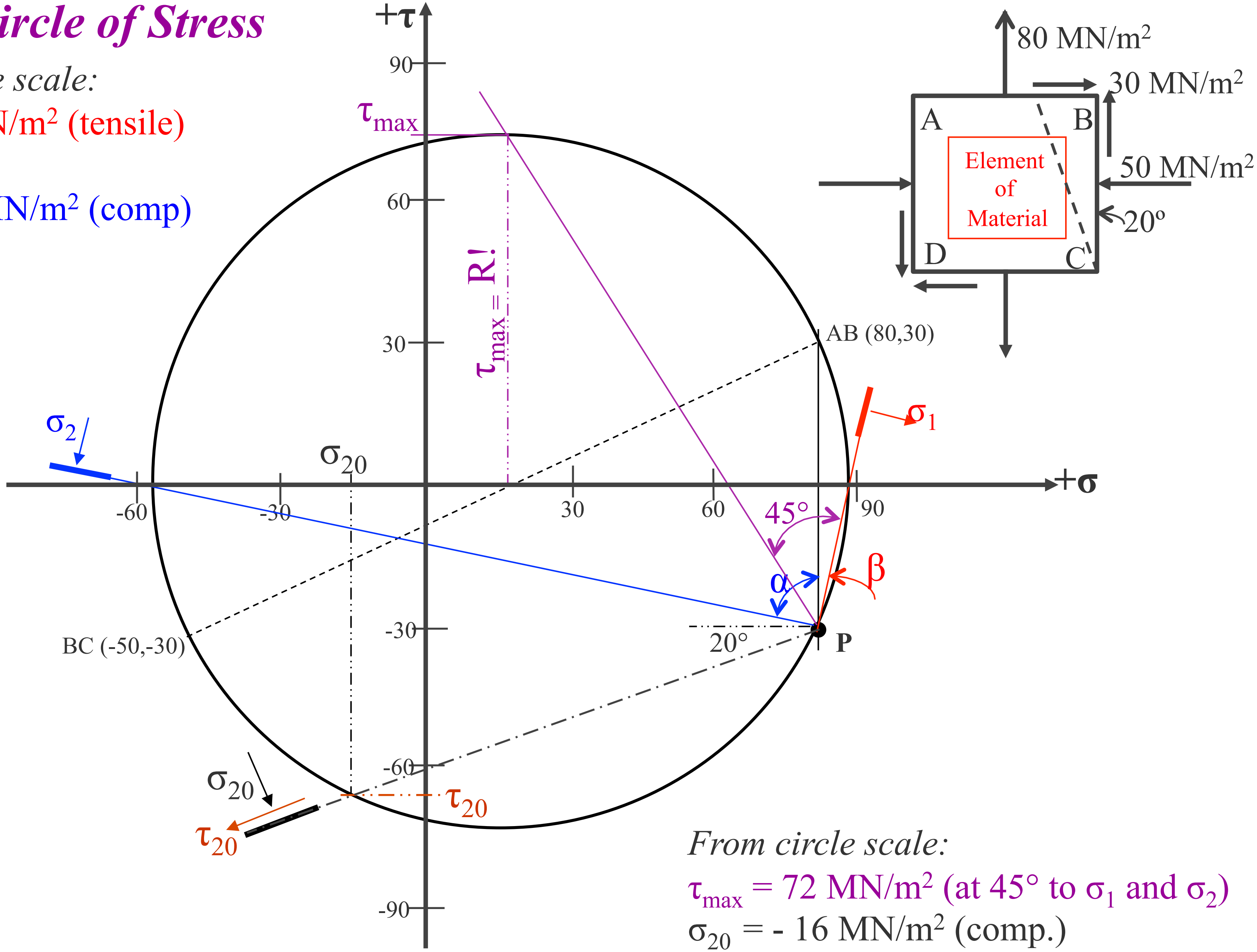
From circle scale:

σ_1 = 87 MN/m^2 (tensile)

β = 13°

σ_2 = - 57 MN/m^2 (comp)

α = 77°



From circle scale:

τ_{max} = 72 MN/m^2 (at 45° to σ_1 and σ_2)

σ_{20} = - 16 MN/m^2 (comp.)

τ_{20} = - 65 MN/m^2

Analytical Solution

(i)

From element: σ_x = -50MN/m^2 (comp), σ_y = 80MN/m^2 (tensile) and τ_{xy}= 30MN/m^2 (ACW)

Major principal stress - σ_1 :

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \frac{-50+80}{2} + \sqrt{\left(\frac{-50-80}{2}\right)^2 + 30^2}$$

= 86.59 MN/m^2
(tension)

Minor principal stress - σ_2 :

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \frac{-50+80}{2} - \sqrt{\left(\frac{-50-80}{2}\right)^2 + 30^2}$$

= -56.59 MN/m^2
(compression)

(ii) Max. shear stress – τ_{max} :

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \frac{\sigma_1 - \sigma_2}{2} = \frac{86.59-(-56.59)}{2} = 71.59 \text{ MN/m}^2$$

(iii) Normal stress on 20º plane - σ_{20} i.e. θ= 20°:

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau \sin 2\theta = \frac{-50+80}{2} + \frac{-50-80}{2}\cos 2(20) + 30\sin 2(20)$$

i.e. σ_θ = -15.51 MN/m² (compressive)

(iv) Shear stress on 20º plane - τ_{20} i.e. θ= 20°:

$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau\cos 2\theta = \frac{-50-80}{2}\sin 2(20) - 30\cos 2(20)$$

i.e. τ_θ = -64.76 MN/m²