Worked Example
Thin Cylinders and Shells

An air vessel of length 2 m has an external diameter of 400 mm is manufactured from 10 mm thick steel plate as shown in Figure Ex.1. The steel has a Young's modulus of 200 GN/m^2 and a Poisson's ratio of 0.3. The vessel is subjected to an internal pressure of 10 MN/m^2. Determine:

(a) the hoop and longitudinal stresses set-up in the vessel material;
(b) the increase in the internal diameter;
(c) the increase in the vessel length

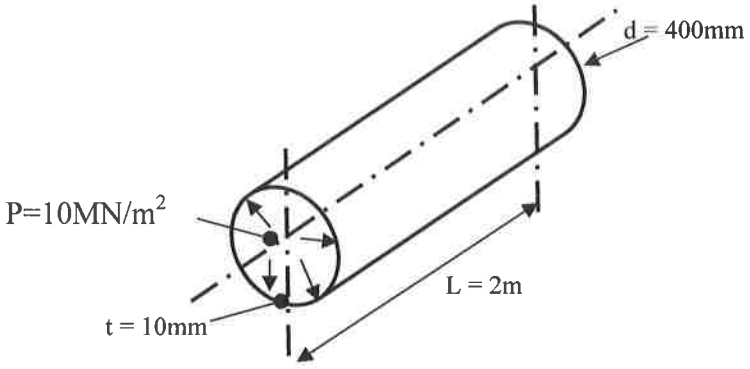

Figure Ex.1

Solution

(a) Hoop and Longitudinal Stresses:

$$\sigma_H = \frac{pd}{2t} = \frac{(10x10^6)x0.38}{2x0.01} = \mathbf{190\ MN/m^2}$$

$$\sigma_L = \frac{pd}{4t} = \frac{(10x10^6)x0.38}{4x0.01} = \mathbf{95\ MN/m^2}$$

(b) Increase in Internal Diameter:

$$\therefore \quad \Delta d = \frac{pd^2}{4tE}(2-\nu) = \frac{10x10^6\,x0.38^2}{4x0.01x(200x10^9)}(2-0.3) = 0.00031\ \text{m} = \mathbf{0.31\ mm}$$

(c) Increase in Length:

$$\therefore \quad \Delta L = \frac{pdL}{4tE}(1-2\nu) = \frac{10x10^6\,x0.38x2}{4x0.01x(200x10^9)}(1-2(0.3)) = 0.00038\ \text{m} = \mathbf{0.38\ mm}$$