School of Engineering & Built Environment

African Leadership College Notes

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Module: Engineering Design and Analysis 2

Spur Gear Summary

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Original Epicyclic and Power Screw notes prepared by G.K. Vijayaraghavan (2006), Original Clutch, Belt and Brake Systems by Dr M. Macdonald, Updated notes prepared by A. Cowell (2017)

2.1. Spur Gear Force Analysis

Due to the fact that spur gears have straight teeth, they are the easiest of the gears to analyse and have just two force components:

i) The tangential force, $F_t = \frac{T}{r}$, and; (2.1)

ii) The radial force, $F_r = F_t \tan\phi$ (2.2)

Power:

$$P = T \times \omega \qquad (2.3)$$

Torque:

$$T = \frac{60 \times P}{2\pi N} \qquad (2.4)$$

Pitch-line velocity:

$$v = \frac{\pi d N}{60} \qquad (2.5)$$

Remembering also that $\omega = v / r = 2v / d$, and rewriting Equation (2.1) as:

$$T = \frac{d}{2} \times F_t \qquad (2.6)$$

Equations (2.3) and (2.6) can be combined to give:

$$P = \frac{d}{2} \times F_t \times v \times \frac{2}{d}$$

$$\Rightarrow P = F_t \times v$$

$$\Rightarrow F_t = \frac{P}{v}$$

2.2. Stresses in Gear Teeth

The Lewis Equation – Strength of Gear Teeth

Lewis equation:

$$F_t = \sigma_t b Y p_c \qquad (2.15)$$

Barth's equation:

$$\begin{aligned} \text{Allowable}\,\sigma_t &= \sigma_0 \left(\frac{3}{3+v} \right) && \text{for } v \text{ less than } 10 \text{ ms}^{-1} \\ &= \sigma_0 \left(\frac{6}{6+v} \right) && \text{for } v \text{ 10 to 20 ms}^{-1} \\ &= \sigma_0 \left(\frac{5.6}{5.6+\sqrt{v}} \right) && \text{for } v \text{ greater than 20 ms}^{-1} \end{aligned} \qquad (2.16)$$

Modified Lewis equation:

$$F_t = \sigma_t kYp_c^2 = \sigma_t \pi^2 kYm^2 \quad (2.17)$$

Where:

$$b = kp_c \quad k \leq 4 \quad (2.18)$$

For a known pitch diameter:

$$\frac{1}{Ym^2} = \frac{\sigma_t k\pi^2}{F_t} \quad (2.19)$$

If the pitch diameter is unknown:

$$\sigma_t = \frac{2T_p}{\pi^2 kYm^3 z_p} \leq Equation(2.16) \quad (2.20)$$

Buckingham Equations – Design for Dynamic Tooth Load and Tooth Wear

The Buckingham equation:

$$F_d = \frac{21v(bC + F_t)}{21v + \sqrt{(bC + F_t)}} + F_t \quad (2.21)$$

The "allowable endurance load" (Hall et al, 1980):

$$F_0 = \sigma_0 bYp_c \quad (2.22)$$

If $F_d < F_0$ then the dynamic load is acceptable.

The wear load to avoid excessive contact stress is given by (Hall et al, 1980):

$$F_w = D_p bKQ \quad (2.22)$$

Where:

$$Q = 2z_g / (z_p + z_g) \quad (2.23)$$

And, the stress factor for fatigue, K (Nm^{-2}) is:

$$K = \frac{s_{es}^2 \sin\phi (1/E_p + 1/E_g)}{1.4} \quad (2.24)$$

The value for s_{es} (in MNm^{-2}) can be estimated from:

$$s_{es} = 2.75(BHN) - 70 \quad (2.25)$$

2.3. Nomenclature for Spur Gears

Symbol	Description	Unit
b	Face width	m
BHN	Brinell Hardness Number	-
C	Deformation Factor	kNm^{-1}
D	Pitch circle diameter	m
E	Elastic (Young's) modulus	Nm^{-2}
F_0	Allowable endurance load	N
F_d	Dynamic force	N
F_r	Radial force	N
F_t	Tangential (Transmitted) force	N
F_w	Allowable wear load	N
h	Tooth height	m
I	Second moment of area	m^4
i	Velocity ratio	-
K	Stress factor for fatigue	Nm^{-2}
k	Ratio of face width to circular pitch	-
M	Moment on gear tooth	Nm
m	Module (module pitch)	m
N	Rotational speed	rpm
P	Power	W
p_c	Circular pitch	m
p_d	Diameter pitch	m
Q	Geometry factor for tooth dynamic strength analysis	-
r	Gear radius	m
s_{es}	Surface endurance limit of a gear pair	Nm^{-2}
T	Torque	Nm
t	Tooth width	m
v	Pitch line velocity	ms^{-1}
W	Tooth force	N
W_n	Normal tooth force	N
W_r	Radial tooth force	N
Y	Tooth 'Form Factor'	-
y	Distance from neutral axis	m
z	Number of teeth	-
ϕ	Pressure angle	degrees
σ_0	Endurance strength for released loading corrected for average stress concentration values of the material	Nm^{-2}
σ_b	Bending stress	Nm^{-2}
σ_t	Stress at the base of the tooth profile	Nm^{-2}
ω	Rotational velocity	$rads^{-1}$
	Subscripts	
g	Gear	
p	Pinion	