5.4. SOLVED PROBLEMS

Problem 1

An epicyclic gear consists of three gears A, B and C as shown in Figure 5.3. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates anticlockwise about the centre of A at 18 rpm. If the gear A is fixed, determine the speed of gear B and C.

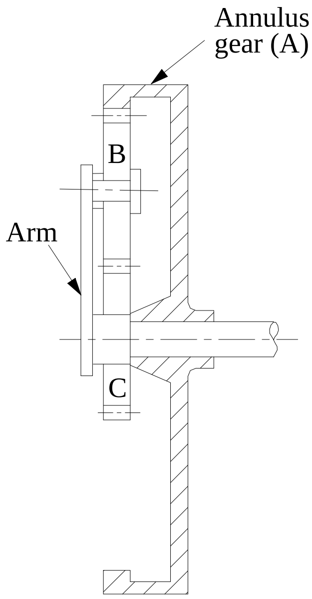


Figure 5.3

Given:

$z_A = 72$; $z_C = 32$; speed of arm = 18 rpm.

Solution:

Considering the relative motion of rotation as shown in table

Step no	***Conditions of motions***	***Revolutions of elements***			
		Arm EF	***Gear C***	***Gear B***	***Gear A***
1.	Arm fixed, gear C rotates through + 1 revolutions (i.e. Anti-clockwise)	0	+1	$-\frac{z_C}{z_B}$	$-\frac{z_C}{z_B}\times\frac{z_B}{z_A}=-\frac{z_C}{z_A}$
2.	Arm fixed, gear C rotates through + x revolutions (i.e. Anti-clockwise)	0	$+x$	$-x\times\frac{z_C}{z_B}$	$-x\times\frac{z_C}{z_A}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\frac{z_C}{z_B}$	$y-x\frac{z_C}{z_A}$

1. Speed of gear C

We know that the speed of the arm is 18 rpm therefore,

$$y = 18 \text{ rpm.}$$

And the gear A is fixed, therefore

$$y-x\frac{z_C}{z_A} = 0 \text{ or } \quad 18-x\frac{32}{72} = 0$$

$$x = 18\times\frac{72}{32} = 40.5 \text{ rpm}$$

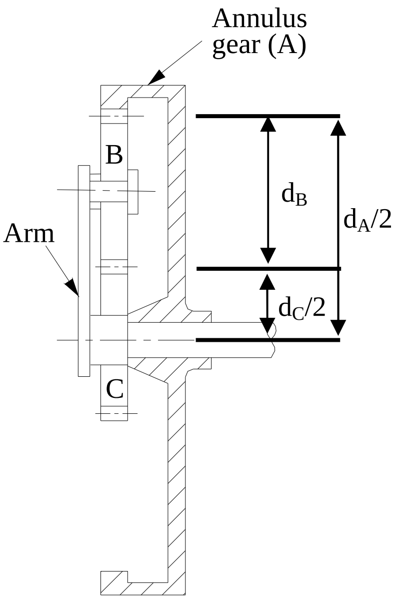
Speed of gear C = $x + y$ = 40.5 + 18

= + 58.5 rpm

= **58.5 rpm anticlockwise. Ans.** ∗

2. Speed of gear B

Let d_A, d_B and d_C be the pitch circle diameter of gears A, B and C respectively. Therefore from the geometry of Figure 5.3:



$$d_B + \frac{d_C}{2} = \frac{d_A}{2} \qquad \text{or} \qquad 2\,d_B + d_C = d_A$$

Since the number of teeth are proportional to their pitch circle diameter (d = m.z), therefore

$$2\,z_B + z_C = z_A \qquad \text{or} \qquad 2 \times z_B + 32 = 72 \qquad \text{or} \qquad z_B = 20$$

$$\text{Speed of gear B} = y - x\frac{z_C}{z_B} = 18 - \left(40.5 \times \frac{32}{20}\right) = -\,46.8 \text{ rpm}$$

= **46.8 rpm clockwise**. **Ans.** ∗

Problem 2

In an epicyclic gear train shown in Figure 5.4, the internal gears A and B and compound gears C and D rotate independently about the axis O. The gears E and F rotate on pins fixed to the arm G, E gear with A and C, and F gears with B and D. All the gears have the same module and the number of teeth is:

$z_C = 28;\ z_D = 26;\ z_E = 18;\ z_F = 28.$

1. ***Find the number of teeth on A and B;***
2. ***If the arm G makes 100 rpm clockwise and A is fixed, find the speed of B; and***
3. ***If the arm G makes 100 rpm clockwise and gear A makes 10 rpm anticlockwise; find the speed of gear B.***

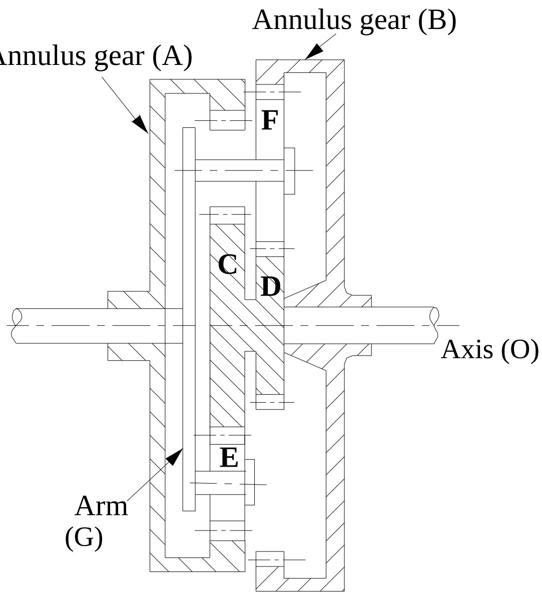


Figure 5.4

Given:

$z_C = 28$; $z_D = 26$; $z_E = 18$; $z_F = 28$

Solution:

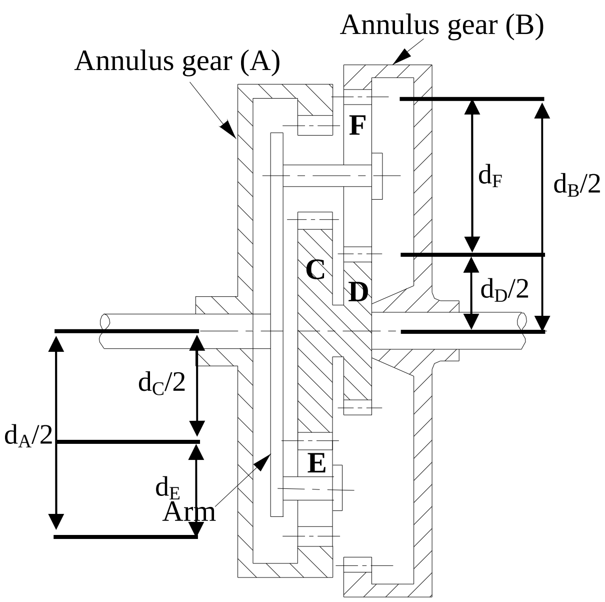
1. Number of teeth on gears A and B

Let

z_A $\Rightarrow$ Number of teeth on gear A and

z_B $\Rightarrow$ Number of teeth on gear B

If d_A, d_B, d_C, d_D, d_E and d_F are the pitch circle diameters of gears A, B, C, D, E and F respectively, then from the geometry of Figure 5.4:



$$d_A = d_C + 2\,d_E$$

And

$$d_B = d_D + 2\,d_F$$

Since the number of teeth are proportional to their pitch circle diameters, for the same module therefore

$z_A = z_C + 2\,z_E = 28 + 2\times 18 =$ **64** **Ans.** ∗

$z_B = z_D + 2\,z_F = 26 + 2\times 28 =$ **82** **Ans.** ∗

2. Speed of gear B when arm G makes 100 rpm clockwise and gear A is fixed

First of all, the table motion is drawn as given below

S. no	Conditions of motion	Revolutions of motions					
		Arm G	Gear A	Gear E	Compound gear C-D	Gear F	Gear B
1.	Arm fixed gear A rotates through +1 revolution	0	+1	$+\frac{z_A}{z_E}$	$-\frac{z_A}{z_E}\times\frac{z_E}{z_C}$ $=-\frac{z_A}{z_C}$	$+\frac{z_A}{z_C}\times\frac{z_D}{z_F}$	$+\frac{z_A}{z_C}\times\frac{z_D}{z_F}\times\frac{z_F}{z_B}$ $=+\frac{z_A}{z_C}\times\frac{z_D}{z_B}$
2.	Arm fixed gear A rotates through + x revolutions	0	+ x	$+x\times\frac{z_A}{z_E}$	$-x\times\frac{z_A}{z_C}$	$+x\times\frac{z_A}{z_C}\times\frac{z_D}{z_F}$	$+x\times\frac{z_A}{z_C}\times\frac{z_D}{z_B}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y	+ y	+ y	+ y
4.	Total motion	+ y	$x + y$	$y+x\times\frac{z_A}{z_E}$	$y-x\times\frac{z_A}{z_C}$	$y+x\times\frac{z_A}{z_C}\times\frac{z_D}{z_F}$	$y+x\times\frac{z_A}{z_C}\times\frac{z_D}{z_B}$

Since the arm G makes 100 rpm clockwise, therefore from the fourth row of the table,

$y = -100$

Also, the gear is fixed, therefore from the fourth row of the table,

$x + y = 0$ or $x = -y = 100$

Speed of the gear B

$$= y + x\left\{\left(\frac{z_A}{z_C}\right)\times\left(\frac{z_D}{z_B}\right)\right\} = -100 + 100\times\left\{\left(\frac{64}{28}\right)\times\left(\frac{26}{82}\right)\right\} = -27.5 \text{ rpm}$$

= **27.5 rpm clockwise** **Ans.** ∗

3. Speed of gear B when arm G makes 100 rpm clockwise and gear A makes 10 rpm anticlockwise

Since the arm G makes 100 rpm clockwise, therefore from the fourth row of the table

$y = -100$ rpm

Also the gear A makes 10 rpm counterclockwise, therefore from the fourth row of the table,

$x + y = 10$ or $x = 10 - y = 10 + 100 = 110$ rpm

Speed of gear B

$$= y + x\left\{\left(\frac{z_A}{z_C}\right)\times\left(\frac{z_D}{z_B}\right)\right\} = -100 + 110\left\{\left(\frac{64}{28}\right)\times\left(\frac{26}{82}\right)\right\} = -20.3 \text{ rpm}$$

= **20.3 rpm clockwise** **Ans.** ∗

Problem 3

Two shafts A and B are co-axial. A gear C (50 teeth) is rigidly mounted on shaft A. Compound gear D-E gears with C and an internal gear G. D has 20 teeth and gears with C and E has 35 teeth and gears with an internal gear G. The gear G is fixed and is concentric with the shaft axis. The compound gear D-E is mounted on a pin which projects from an arm keyed to the shaft B. Find the number of teeth on internal gear G assuming that all gears have the same module. If the shaft A rotates at 110 r.p.m., find the speed of shaft B.

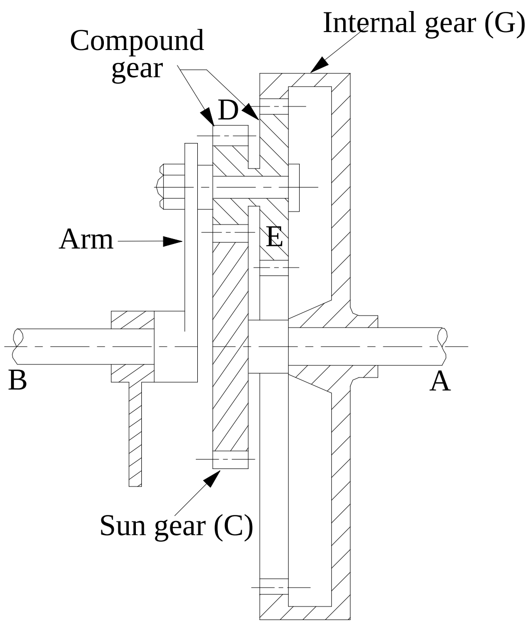


Figure 5.5

Given:

$z_C = 50$; $z_D = 20$; $z_E = 35$; $N_A = 110$ r.p.m.

Solution:

1. *Number of teeth on internal gear G*

Let d_C, d_D, d_E and d_G be the pitch circle diameters of gear C, D, E and G respectively. From the geometry of the figure:

$$\frac{d_G}{2} = \frac{d_C}{2} + \frac{d_D}{2} + \frac{d_E}{2}$$

$$d_G = d_C + d_D + d_E$$

Let z_C, z_D, z_E and z_G be the number of teeth on gears C, D, E and G respectively. Since all the gears have the same module, therefore number of teeth is proportional to their pitch circle diameters.

$\therefore\ z_G = z_C + z_D + z_E$ = 50 + 20 + 35 = **105** **Ans.** ∗

2. Speed of shaft B

The table of motions is given below:

S. No.	Conditions of motion	Revolutions of elements			
		Arm	Gear C (or shaft A)	Compound gear D-E	Gear G
1.	Arm fixed-gear C rotates through +1 revolution.	0	+1	$-\frac{z_C}{z_D}$	$-\frac{z_C}{z_D}\times\frac{z_E}{z_G}$
2.	Arm fixed-gear C rotates through +*x* revolutions.	0	$+x$	$-x\times\frac{z_C}{z_D}$	$-x\times\frac{z_C}{z_D}\times\frac{z_E}{z_G}$
3.	Add +*y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x + y$	$y - x\times\frac{z_C}{z_D}$	$y - x\times\frac{z_C}{z_D}\times\frac{z_E}{z_G}$

Since the gear G is fixed, therefore from the fourth row of the table,

$$y - x\times\frac{z_C}{z_D}\times\frac{z_E}{z_G} = 0 \text{ or } y - x\times\frac{50}{20}\times\frac{35}{105} = 0$$

$$\therefore\quad y - \frac{5}{6}x = 0$$

$$\therefore\quad y = \frac{5}{6}x \qquad \text{(i)}$$

Since the gear C is rigidly mounted on shaft A, therefore speed of gear C and shaft A is same. We know that speed of shaft A is 110 r.p.m., therefore from the fourth row of the table.

$x + y = 110$ (ii)

From equations (i) and (ii);

$$x+\frac{5}{6}x=110 \text{ which } \Rightarrow x=110\times\frac{6}{11}$$

$\therefore$ x = 60 and y = 50

$\therefore$ Speed of the shaft, B = Speed of arm = +y = **50r.p.m. anticlockwise Ans.** ∗

Problem 4

An internal gear B with 80 teeth is keyed to a shaft F. A fixed internal gear C with 82 teeth is concentric with B. A compound gear D-E gears with the two internal gears, D has 28 teeth and gears with C, while E gears with B. The compound gears revolve freely on a pin which projects from a disc keyed to a shaft A, co-axial with F. If the gears have the same pitch and the shaft A makes 800 r.p.m. anticlockwise, what is the speed of the shaft F? Sketch the arrangement.

Given:

z_B = 80; z_C = 82; z_D = 28; N_A = 800 r.p.m

Solution:

First of all, let us find out the number of teeth on gear E (z_E). Let d_B, d_C, d_D and d_E be the pitch circle diameter of gears B, C, D and E respectively. From the geometry of Figure 5.6:

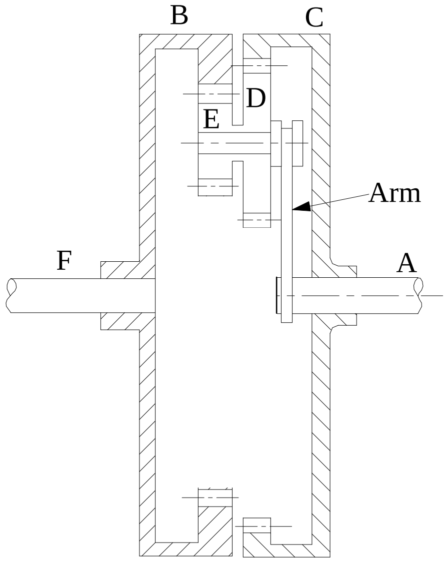


Figure 5.6

$$d_B/2=d_C/2-d_D/2+d_E/2$$

$$d_E=d_B+d_D-d_C$$

Since the number of teeth are proportional to their pitch circle diameters for the same pitch, therefore:

$z_E = z_B + z_D - z_C$ = 80 + 28 – 82 = 26

The table of motions is given below:

Step No.	Conditions of motion	Revolutions of elements			
		Arm (shaft A)	Gear B (or shaft F)	Compound gear D-E	Gear C
1.	Arm fixed-gear B rotated through +1 revolution (i.e. 1 revolution anticlockwise)	0	+1	$+\frac{z_B}{z_E}$	$+\frac{z_B}{z_E}\times\frac{z_D}{z_C}$
2.	Arm fixed-gear B rotated through + x revolution.	0	$+x$	$x\times\frac{z_B}{z_E}$	$+x\frac{z_B}{z_E}\times\frac{z_D}{z_C}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y+x\times\frac{z_B}{z_E}$	$y+x\frac{z_B}{z_E}\times\frac{z_D}{z_C}$

Since the gear C is fixed, therefore from the fourth row of the table,

$$y+x\times\frac{z_B}{z_E}\times\frac{z_D}{z_C}=0 \quad \text{or} \quad y+x\times\frac{80}{26}\times\frac{28}{82}=0$$

$\therefore$ $y + 1.05x = 0 \quad \Rightarrow x = -y/1.05$ (i)

Also, the shaft A (or the arm) makes 800 r.p.m. anticlockwise, therefore from the fourth row of the table.

y = 800 (ii)

From equations (i) and (ii)

x = –762

$\therefore$ Speed of shaft F = Speed of gear B = $x + y$ = –762 + 800 = +38r.p.m.

= **38.r.p.m. (anticlockwise) Ans.** ∗

Problem 5

The epicyclic gear train shown in Figure 5.7 is driven through input shaft G. Gears A, B, C, D have 40, 20, 60 and 40 teeth respectively, casing E is normally kept stationary. Determine:

1. ***the number of teeth on casing E;***
2. ***the speed and direction of rotation of input shaft G for an output speed of 950 r.p.m in the clockwise direction;***
3. ***the speed and direction of rotation of output shaft H when input shaft G rotates clockwise at a speed of 420 r.p.m and casing E rotates anticlockwise at 160 r.p.m.***

Given:

N_H = -950 r.p.m; z_A= 40; z_B = 20; z_C = 60; z_D = 40;

Solution:

First of all, let us find out the number of teeth on casing E (z_E). Let d_A, d_B, d_C, and d_D be the pitch circle diameter of gears A, B, C, and D respectively. From the geometry of Figure 5.7.

$$d_E = d_A + 2d_B + (d_C - d_D)$$

Since the number of teeth are proportional to their pitch circle diameters for the same pitch, therefore:

$z_E = z_A + 2z_B + (z_C - z_D)$ = 40 + (2×20) + 60 – 40 = **100 teeth** **Ans.** ∗

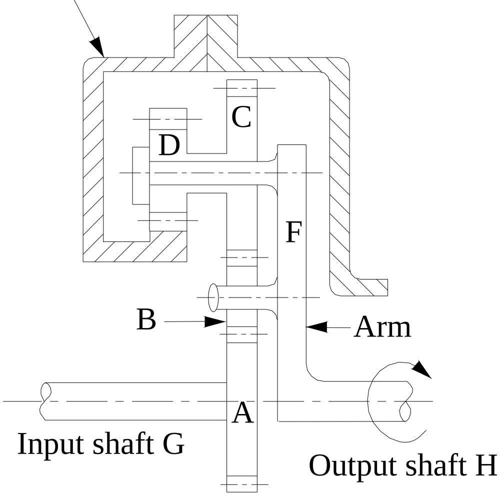


Figure 5.7

First of all, the table motion is drawn as given below

S. no	Conditions of motion	Revolutions of motions				
		Arm F	Gear A	Gear B	Compound gear C-D	Casing E
1.	Arm fixed gear A rotates through +1 revolution	0	1	$-\frac{z_A}{z_B}$	$\frac{z_A}{z_B}\times\frac{z_B}{z_C}=\frac{z_A}{z_C}$	$-\frac{z_A}{z_C}\times\frac{z_D}{z_E}$
2.	Arm fixed gear A rotates through + x revolutions	0	x	$-x\times\frac{z_A}{z_B}$	$x\times\frac{z_A}{z_C}$	$-x\times\frac{z_A}{z_C}\times\frac{z_D}{z_E}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$
4.	Total motion	y	$x+y$	$y-x\times\frac{z_A}{z_B}$	$y+x\times\frac{z_A}{z_C}$	$y-x\times\frac{z_A}{z_C}\times\frac{z_D}{z_E}$

2. Speed of input shaft G

Since the gear E is stationary, therefore from the fourth row of the table,

$$y-x\times\frac{z_A}{z_C}\times\frac{z_D}{z_E}=0 \quad \text{or} \quad y-x\times\frac{40}{60}\times\frac{40}{100}=0$$

$\therefore 0.27\,x = y$ …..(i)

Also, the output shaft H (or the arm) makes 950 r.p.m clockwise. Therefore from the fourth row of the table:

y = -950 ……(ii)

From equations (i) and (ii)

x = -3562.5

$\therefore$ Speed of input shaft G = Speed of gear A = $x + y$ = -3562.5 - 950

= **- 4512.5 r.p.m. (Clockwise) Ans.** ∗

3. speed and direction of rotation of output shaft H

Since the input shaft G rotates clockwise at a speed of 420 r.p.m and casing E rotates anticlockwise at 160 r.p.m.

$x + y = -420$ …..(iii)

$$y-x\times\frac{z_A}{z_C}\times\frac{z_D}{z_E}=160$$

$$y - x \times \frac{40}{60} \times \frac{40}{100} = 160$$

$\therefore$ y - 0.27 x = 160 …..(iv)

From equations (iii) and (iv)

x = -457.9

y = 37.9

$\therefore$ Speed of output shaft H = Speed of arm F = + y = **37.9 (Anticlockwise) Ans.** ∗

Problem 6

The epicyclic gear train shown in Figure 5.8 is driven with input shaft A. Gears A, B, and C have 48, 30 and 18 teeth respectively. Gears D and F, and E and G are identical having 30 and 18 teeth respectively and casing H is kept stationary. Determine;

1. ***the number of teeth on casing H;***
2. ***the speed and direction of rotation of output shaft K for an input speed of 1000 rev/min in the clockwise direction;***
3. ***the speed and direction of rotation of output shaft K when the input shaft A rotates clockwise at a speed of 1000 rev/min and casing H rotates anticlockwise at a speed of 180 rev/min.***

Given:

N_A = 1000 r.p.m; z_A= 48; z_B = 30; z_C = 18; z_D = 30; z_E= 18; z_F = 30; z_G = 18;

Solution:

1. *Number of teeth on casing H(z_H)*

Let d_A, d_B, d_C, d_D, d_E, d_F, and d_G be the pitch circle diameter of gears A, B, C, D, E, F, and G respectively. From the geometry of Figure 5.8:

$$d_H = d_A + d_B + d_c + d_D - d_E - d_F + d_G$$

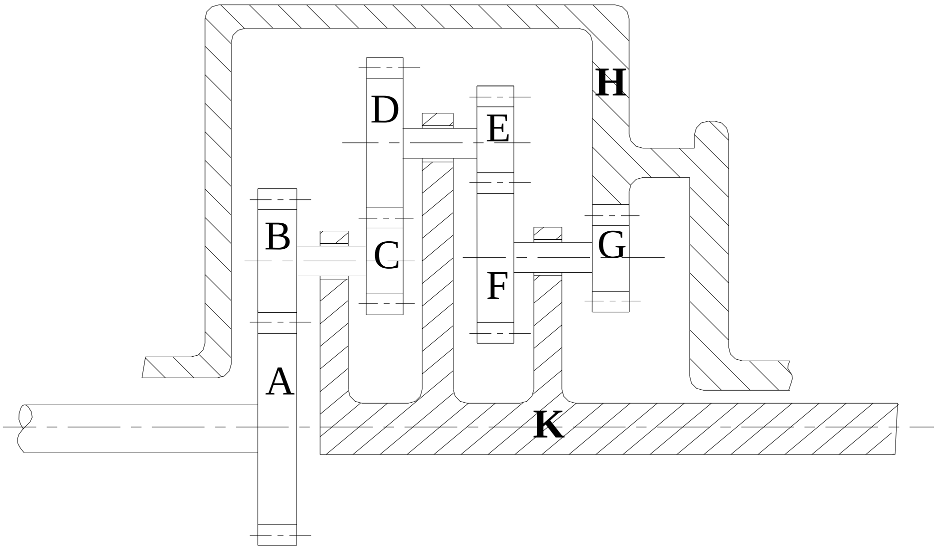


Figure 5.8

Since the number of teeth are proportional to their pitch circle diameters for the same pitch, therefore

$$z_H = z_A + z_B + z_c + z_D - z_E - z_F + z_G$$

= 48+30+18+30–18– 30+18 =**96 teeth** **Ans.** ∗

Number of teeth on casing H is 96 teeth

First of all, the table of motion is drawn as given below

Conditions of motion	Revolutions of motions					
	Arm K	Gear A	Gear B-C	Gear D-E	Gear F-G	Casing H
Arm fixed- gear A rotates through +1 revolution	0	1	$-\frac{z_A}{z_B}$	$\frac{z_A}{z_B}\times\frac{z_C}{z_D}$	$-\frac{z_A}{z_B}\times\frac{z_C}{z_D}\times\frac{z_E}{z_F}$	$-\frac{z_A}{z_B}\times\frac{z_C}{z_D}\times\frac{z_E}{z_F}\times\frac{z_G}{z_H}$
Arm fixed gear A rotates through + x revolutions	0	x	$-x\times\frac{z_A}{z_B}$	$x\frac{z_A}{z_B}\times\frac{z_C}{z_D}$	$-x\frac{z_A}{z_B}\times\frac{z_C}{z_D}\times\frac{z_E}{z_F}$	$-x\frac{z_A}{z_B}\times\frac{z_C}{z_D}\times\frac{z_E}{z_F}\times\frac{z_G}{z_H}$
Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$	$+y$
Total motion	y	$x+y$	$y-x\times\frac{z_A}{z_B}$	$y+x\frac{z_A}{z_B}\times\frac{z_C}{z_D}$	$y-x\frac{z_A}{z_B}\times\frac{z_C}{z_D}\times\frac{z_E}{z_F}$	$y-x\frac{z_A}{z_B}\times\frac{z_C}{z_D}\times\frac{z_E}{z_F}\times\frac{z_G}{z_H}$

2. Speed and direction of rotation of output shaft K

Since the casing H is stationary, therefore from the fourth row of the table,

$$y - x\frac{z_A}{z_B} \times \frac{z_C}{z_D} \times \frac{z_E}{z_F} \times \frac{z_G}{z_H} = 0 \quad \text{or} \quad y - x\frac{48}{30} \times \frac{18}{30} \times \frac{18}{30} \times \frac{18}{96} = 0$$

$\therefore$ y - 0.108 x = 0 …..(i)

Also, the input shaft A makes 1000 r.p.m clockwise, therefore from the fourth row of the table.

$x + y = -1000$ ……(ii)

From equations (i) and (ii)

x = - 902.5

y = - 97.47

$\therefore$ Speed of output shaft K = Speed of arm K = y = **–97.47r.p.m. (Clockwise) Ans.** ∗

3. Speed and direction of rotation of output shaft K when the input shaft A rotates clockwise at a speed of 1000 rev/min and casing H rotates anticlockwise at a speed of 180 rev/min

Since the casing H makes 180 r.p.m anticlockwise, therefore from the fourth row of the table,

$$y - x\frac{z_A}{z_B} \times \frac{z_C}{z_D} \times \frac{z_E}{z_F} \times \frac{z_G}{z_H} = 180 \quad \text{or} \quad y - x\frac{48}{30} \times \frac{18}{30} \times \frac{18}{30} \times \frac{18}{96} = 180$$

$\therefore$ y - 0.108 x = 180 …..(iii)

Also, the input shaft A makes 1000 r.p.m clockwise, therefore from the fourth row of the table.

$x + y = -1000$ ……(iv)

From equations (iii) and (iv)

x = -1065

y = 65

$\therefore$ Speed of output shaft K = Speed of arm K = y = **65r.p.m. (Anticlockwise) Ans.** ∗

Problem 7

An epicyclic gear train used for reduction gearing is shown in Figure 5.9. It consists of sun gear B on the input shaft, planet gear C carried on arm K on the output shaft, and an annular gear D within the casing E. The casing can either be held fixed, or connected directly to the input shaft through the spur gear drive AFGHE. The number of teeth on each gear is given below:

Gear	***A***	***B***	***C***	***D***	***F***	***G***	***H***
Teeth	***48***	***48***	***24***	***96***	***40***	***48***	***48***

The input shaft rotates at 450 r.p.m. in clockwise direction. Determine:

1. ***the number of teeth on casing E (i.e., external surface of the casing);***
2. ***the speed and direction of rotation of output shaft K when gears FGH are disengaged and D is held against rotation***
3. ***the speed and rotation of output shaft K when gears FGH are engaged with input shaft A.***

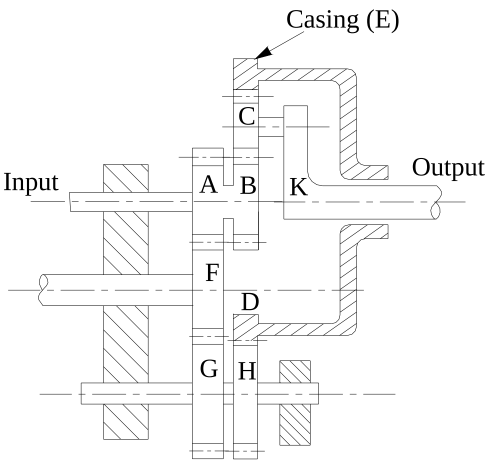


Figure 5.9

Given:

N_A = 450 r.p.m; z_A= 48; z_B = 48; z_C = 24; z_D = 96; z_F = 40; z_G = 48; z_H= 48;

Solution:

1. *Number of teeth on external surface of the casing E(z_E)*

Let $d_{A,}$ d_B, d_C, $d_{D,}$ d_E, d_F, and d_G be the pitch circle diameter of gears A, B, C, D, E, F, and G respectively. From the geometry of Figure 5.9:

$$d_E = d_A + 2d_F + d_G - d_H$$

Since the number of teeth are proportional to their pitch circle diameters for the same pitch, therefore:

$$z_E = z_A + 2z_F + z_G - z_H$$

$= 48+(2\times40)+48–48=$**128 teeth** **Ans.** ∗

Number of teeth on external surface of the casing E is 128 teeth.

First of all, the table of motion is drawn as given below:

Let us consider the gears A, B, C, D and arm K. Then the table of motion is given as

Conditions of motion	Revolutions of motions			
	Arm K	Gear A-B	Gear C	Gear D
Arm fixed- gear A rotates through +1 revolution	0	1	$-\frac{z_B}{z_C}$	$-\frac{z_B}{z_C}\times\frac{z_C}{z_D} = -\frac{z_B}{z_D}$
Arm fixed gear A rotates through + x revolutions	0	x	$-x\times\frac{z_B}{z_C}$	$-x\frac{z_B}{z_D}$
Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$
Total motion	y	$x+y$	$y-x\times\frac{z_B}{z_c}$	$y-x\frac{z_B}{z_D}$

Now, let us consider the gears A, F, G, H, E and arm K. Then the table of motion is given as:

Conditions of motion	Revolutions of motions				
	Arm K	Gear A	Gear F	gear G-H	casing E
Arm fixed- gear A rotates through +1 revolution	0	1	$-\frac{z_A}{z_F}$	$\frac{z_A}{z_F}\times\frac{z_F}{z_G} = \frac{z_A}{z_G}$	$-\frac{z_A}{z_G}\times\frac{z_H}{z_E}$
Arm fixed gear A rotates through + x revolutions	0	x	$-x\times\frac{z_A}{z_F}$	$x\frac{z_A}{z_G}$	$-x\frac{z_A}{z_G}\times\frac{z_H}{z_E}$
Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$
Total motion	y	$x+y$	$y-x\times\frac{z_A}{z_F}$	$y+x\frac{z_A}{z_G}$	$y-x\frac{z_A}{z_G}\times\frac{z_H}{z_E}$

2. Speed and direction of rotation of output shaft K

Since the casing D is held against rotation, from the fourth row of the first table;

$$y - x\frac{z_B}{z_D} = 0 \text{ or } y - x\frac{48}{96} = 0$$

$\therefore$ y – 0.5x = 0 …..(i)

Also, the input shaft A makes 450r.p.m clockwise, therefore from the fourth row of the first table:

x + y = – 450……(ii)

From equations (i) and (ii)

x = -300

y = -150

$\therefore$ Speed of output shaft K = Speed of arm K = y = **–150r.p.m. (Clockwise) Ans.** ∗

3. Speed and direction of rotation of output shaft K when the input shaft A rotates clockwise at a speed of 450 rev/min and gears FGH engaged

Speed of gear E = $y - x\frac{z_A}{z_G} \times \frac{z_H}{z_E} = -150 + 300 \times \frac{48}{48} \times \frac{48}{128} = -37.5\, r.p.m$

Speed of gear D = Speed of gear E

$$y - x\frac{z_B}{z_D} = -37.5 \text{ r.p.m.}$$

$$y - x\frac{48}{96} = -37.5$$

$\therefore$ y – 0.5x = –37.5 …..(iii)

Also, the input shaft A makes 450 r.p.m clockwise., therefore from the fourth row of the table.

x + y = – 450 ..…(iv)

From equations (iii) and (iv)

x = -275

y = -175

$\therefore$ Speed of output shaft K = Speed of arm K = y = **–175r.p.m. (Clockwise) Ans.** ∗

Problem 8

The two stage compound epicyclic gear system shown in Figure 5.10 is driven through input shaft A, by an electric motor running at 500 r.p.m in the anticlockwise direction. Gears A, C, D and F have 30, 90, 20 and 80 teeth respectively. Casing C is fixed, and casing F is free to rotate. Shaft D can either be stationary or engaged with A, and thus will rotate at the same speed as A. Determine:

1. ***the number of teeth on gears B and E;***
2. ***the speed and direction of rotation of output shaft G when shaft D is disengaged from A and is stationary;***
3. ***the speed and direction of rotation of output shaft G when shaft D is engaged with shaft A.***

Given:

N_A = 500 r.p.m; z_A= 30; z_C = 90; z_D = 20; z_F = 80;

Solution:

First of all, let us find out the number of teeth on gears B and E (z_B and z_E). Let d_A, d_B, d_C, and d_D be the pitch circle diameter of gears A, B, C, and D respectively. From the geometry of the figure:

$$d_C = d_A + 2d_B$$

Since the number of teeth are proportional to their pitch circle diameters for the same pitch, therefore

$$z_C = z_A + 2z_B$$

$$\therefore z_B = \frac{z_C - z_A}{2} = \frac{90-30}{2}$$ = **30 teeth** **Ans.** ∗

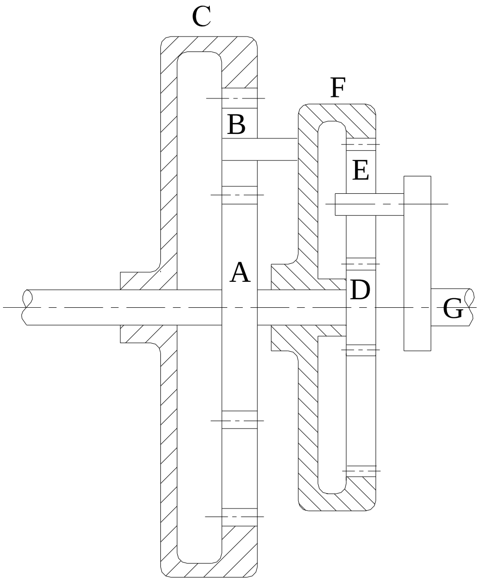


Figure 5.10

Similarly,

$$z_F = z_D + 2z_E$$

$$\therefore z_E = \frac{z_F - z_D}{2} = \frac{80-20}{2} = \textbf{30 teeth} \qquad \textbf{Ans.}$$ ∗

First of all, let us consider the train of gear A, gear B, annular gear C and the annular gear F. Assume annular gear F as an arm.

The table of motion for the train considered (A-B-C-F) is given below.

Table of motions

Step No.	Conditions of motion	Revolutions of elements			
		Arm F	Gear A	Gear B	Gear C
1.	Arm fixed-gear A rotated through +1 revolution (anticlockwise)	0	+1	$-\frac{z_A}{z_B}$	$-\frac{z_A}{z_B}\times\frac{z_B}{z_C}=-\frac{z_A}{z_C}$
2.	Arm fixed-gear A rotated through + *x* revolutions.	0	+*x*	$-x\times\frac{z_A}{z_B}$	$-x\times\frac{z_A}{z_C}$
3.	Add +*y* revolutions to all elements	+*y*	+*y*	+*y*	+*y*
4.	Total motion	+*y*	*x* + *y*	$y-x\times\frac{z_A}{z_B}$	$y-x\times\frac{z_A}{z_C}$

Since the input shaft A rotates at a speed of 500 r.p.m

$x + y = 500$ .....(i)

Assume gear C is stationary

$$y - x \times \frac{z_A}{z_C} = 0$$

$$y - x \times \frac{30}{90} = 0$$

$\therefore$ $y - 0.33\,x = 0$ .....(ii)

From equations (i) and (ii)

$x = 375$

$y = 125$

$\therefore$ Speed of casing F= Speed of arm F = y = **125 r.p.m (Anticlockwise)** **Ans.** ∗

Now, let us consider the train of gear D, gear E, annular gear F and the arm G. The table of motion for the train considered is given below.

Step No.	Conditions of motion	Revolutions of elements			
		Arm G	Gear D	Gear E	Gear F
1.	Arm fixed-gear D rotated through +1 revolution (anticlockwise)	0	+1	$-\frac{z_D}{z_E}$	$-\frac{z_D}{z_E} \times \frac{z_E}{z_F} = -\frac{z_D}{z_F}$
2.	Arm fixed-gear D rotated through + x revolutions.	0	$+x$	$-x \times \frac{z_D}{z_E}$	$-x \times \frac{z_D}{z_F}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x + y$	$y - x \times \frac{z_D}{z_E}$	$y - x \times \frac{z_D}{z_F}$

2. speed and direction of rotation of output shaft G when shaft D is disengaged from A and is stationary

Since the input shaft D is stationary

$x + y = 0$ .....(iii)

Since the speed of the casing F is 125 r.p.m (obtained from previous step)

$$y - x \times \frac{z_D}{z_F} = 125$$

$$y - x \times \frac{20}{80} = 125$$

$y - 0.25\,x = 125$ …..(iv)

From equations (iii) and (iv)

$x = -100$

$y = 100$

$\therefore$ Speed of output shaft G = Speed of arm G = y = **100 r.p.m (Anticlockwise) Ans.** ∗

3. Speed and direction of rotation of output shaft G when shaft D is engaged with shaft A:

Since the input shaft A rotates at a speed of 500 r.p.m anticlockwise

$x + y = 500$ …..(v)

Since the speed of the casing F is 125 r.p.m (obtained from previous step)

$$y - x \times \frac{z_D}{z_F} = 125$$

$$y - x \times \frac{20}{80} = 125$$

$y - 0.25\,x = 125$ …..(vi)

From equations (v) and (vi)

$x = 300$

$y = 200$

$\therefore$ Speed of output shaft G = Speed of arm G = y = **200 r.p.m (Anticlockwise) Ans.** ∗