6.7. Solved Problems *Problem 1*

A turn buckle with right and left hand single start threads is used to couple two wagons. Its thread pitch is 12 mm and its mean diameter 40 mm. The coefficient of friction between the nut and screw is 0.16.

- (i) Determine the work done in drawing the wagons together a distance of 240 mm, against a steady load of 2500 N.
- (ii) If the load increases from 2500 N to 6000 N over the distance of 240 mm, what is the work to be done?

Given data:

$$p = 12 mm$$
, $D_{ms} = 40 mm$, $\mu = \tan \phi = 0.16$, $W = 2500 N$

Solution:

(i) Work done in drawing the wagons together against a steady load of 2500 N

For single start thread l = p

$$\tan \theta = \frac{p}{\pi D_m} = \frac{12}{\pi \times 40} = 0.0955$$
$$\therefore \theta = \tan^{-1} 0.0955 = 5.455^{\circ}$$
$$\tan \phi = 0.16$$

 $: \phi = ton^{-1} 0.16 = 0.00^{\circ}$

$$..\psi = tan 0.10 - 9.09$$

Torque required to overcome friction between the screw and nut (no collar bearing),

T = W tan (
$$\phi + \theta$$
) $\frac{D_{ms}}{2}$ = 2500 tan (9.09 + 5.455) $\times \frac{40}{2}$ = 12972.97 N-mm

A little consideration will show that for one complete revolution of the screwed rod, the wagons are drawn together through a distance equal to 2 p, i.e., $2 \times 12 = 24$ mm. Therefore, in order to draw the wagon together through a distance of 240 mm, the number of turns required is given by

$$N = 240/24 = 10$$

Work done = $T \times 2 \pi N$ = 12972.97×2 π ×10 = 815115.93 N-mm =**815.1 N-m** Ans. ***** (*ii*) Work done in drawing the wagons together when load increases from 2500 N to 6000 N. For an increase in load from 2500 N to 6000 N,

Work done =
$$\frac{815.1(6000 - 2500)}{2500}$$
 = **1141.4 N-m Ans. ***

Problem 2

A 150 mm diameter valve, against which a steam pressure of 2 MNm⁻² is acting, is closed by means of a square threaded screw 50 mm in external diameter with 6 mm pitch. If the coefficient of friction is 0.12, find the torque required to turn the handle.

Given data:

 $D_V = 150 \text{ mm} = 0.15 \text{ m}, \quad p_s = 2 \text{ MNm}^{-2} = 2 \times 10^6 \text{ Nm}^{-2}, \quad D_o = 50 \text{ mm},$ $p = 6 \text{mm}, \quad \mu = \tan \phi = 0.12.$

Solution:

We know that load on the valve,

W = Pressure × Area =
$$p_s \times \frac{\pi}{4} D_v^2 = 2 \times 10^6 \times \frac{\pi}{4} (0.15)^2 = 35343 N$$

Mean diameter of the screw,

$$D_{ms} = D_o - \frac{p}{2} = 50 - \frac{6}{2} = 47mm = 0.047m$$

single start thread $l = n$

For single start thread l = p

$$\tan \theta = \frac{p}{\pi D_{ms}} = \frac{6}{\pi \times 47} = 0.0406$$
$$\therefore \theta = \tan^{-1} 0.0406 = 2.325^{\circ}$$

 $\tan \phi = 0.12$

$$\therefore \phi = \tan^{-1} 0.12 = 6.84^{\circ}$$

Torque required to turn the handle,

T = W tan
$$(\phi + \theta) \frac{D_{ms}}{2}$$

= 35343 tan (6.84 + 2.325) × $\frac{47}{2}$ = **134042.1 N-mm** Ans. *****

Problem 3

A square threaded bolt of root diameter 22.5 mm and pitch 5 mm is tightened by screwing a nut whose mean diameter of bearing surface is 50mm. If the coefficient of friction for the nut and bolt is 0.1 and for nut and bearing surface 0.16, find the force required at the end of a spanner 500 mm long when the load on the bolt is 10 kN.

Given data:

 $D_c = 22.5 \text{ mm}, \quad p = 5 \text{ mm}, \quad D_{mc} = 50 \text{ mm}, \quad \mu_s = \tan \phi = 0.1,$ $\mu_c = 0.16, \quad L = 500 \text{ mm}, \quad W = 10 \text{ kN} = 10000 \text{ N}.$

Let P = Force required at the end of a spanner in Newtons.

We know that mean diameter of the screw,

$$D_{ms} = D_c + \frac{p}{2} = 22.5 + \frac{5}{2} = 25 \, mm$$

$$\therefore \ \tan \theta = \frac{p}{\pi D_{ms}} = \frac{5}{\pi \times 25} = 0.0636$$

$$\therefore \theta = \tan^{-1} 0.0636 = 3.639^{\circ}$$

$$\tan \phi = 0.1$$

$$\therefore \phi = \tan^{-1} 0.1 = 5.71^{\circ}$$

Total torque required,

T = W tan
$$(\phi + \theta) \frac{D_{ms}}{2} + W \mu_c \frac{D_{mc}}{2}$$

= 10000 tan $(5.71 + 3.639) \frac{25}{2} + 10000 \times 0.16 \times \frac{50}{2}$

T = 60579.3 N-mm

We also know that torque required at the end of a spanner,

$$T = P \times L = P \times 500 = 500 P N - mm$$

Equating the two values of torque,

$$500 \times P = 60579.3$$

P = **121.16** N. Ans. *****

Problem 4

A vertical screw with single start square threads of 50 mm mean diameter and 12.5 mm pitch is raised against a load of 10 kN by means of a hand wheel, the boss of which is threaded to act as a nut. The axial load is taken up a thrust collar which supports the wheel boss and has a mean diameter of 60 mm. If the coefficient of friction is 0.15 for the screw and 0.18 for the collar and the tangential force applied by each hand to the wheel is 100 N, find suitable diameter of the hand wheel.

Given data:

$$D_{ms} = 50 \text{ mm},$$
 $p = 12.5 \text{ mm},$ $W = 10000 N,$ $D_{mc} = 60 \text{ mm}$
 $\mu_s = \tan \phi = 0.15,$ $\mu_c = 0.18,$ $P = 100 \text{ N}.$

We know that mean diameter of the screw is 50 mm, therefore,

$$\tan \theta = \frac{p}{\pi D_{ms}} = \frac{12.5}{\pi \times 50} = 0.08$$
$$\therefore \theta = \tan^{-1} 0.08 = 4.574^{\circ}$$
$$\tan \phi = 0.15$$
$$\therefore \phi = \tan^{-1} 0.15 = 8.53^{\circ}$$

Total torque required,

$$T = W \tan(\phi + \theta) \frac{D_{ms}}{2} + W \mu_c \frac{D_{mc}}{2}$$
$$= 10000 \tan(8.53 + 4.574) \frac{50}{2} + 10000 \times 0.18 \times \frac{60}{2}$$

T = 112195.2 N-mm

We know that the torque applied to the hand wheel,

$$T = 2P \times \frac{D_{hw}}{2} = 2 \times 100 \times \frac{D_{hw}}{2} = 100D_{hw} \quad N - mm$$

Equating the two values of torques,

$$100 \times D_{hw} = 112195.2$$

 $D_{hw} = 1121.95 \text{ mm.}$ Ans. *

Problem 5

The pitch of 50 mm mean diameter threaded screw of a screw jack is 12.5 mm. The coefficient of friction between the screw and nut is 0.13. Determine the torque required on the screw to raise a load of 25 kN, assuming the load to rotate with the screw. Determine the ratio of the torque required to raise the load to the torque required to lower the load and also the efficiency of the machine.

Given data:

 $D_{ms} = 50 \text{ mm}, \qquad p = 12.5 \text{mm}, \qquad W = 25000 \text{ N}, \qquad \mu_s = \tan \phi = 0.13$ Solution:

For single start thread l = p

$$\tan \theta = \frac{p}{\pi D_{ms}} = \frac{12.5}{\pi \times 50} = 0.08$$
$$\therefore \theta = \tan^{-1} 0.08 = 4.574^{\circ}$$

 $\tan \phi = 0.13$

$$\therefore \phi = \tan^{-1} 0.13 = 7.4^{\circ}$$

Torque required to raise the screw jack,

$$T_R = W \tan(\phi + \theta) \frac{D_{ms}}{2} = 25000 \tan(7.4 + 4.574) \times \frac{50}{2} = 132551.45 \text{ N-mm}$$

Torque required to lower the screw jack,

$$T_L = W \tan(\phi - \theta) \frac{D_{ms}}{2} = 25000 \tan(7.4 - 4.574) \times \frac{50}{2} = 30851.9 \text{ N-mm}$$

Ratio of the torques required to raise and lower the load:

Ratio of the torques required, $=T_R/T_L = 132551.45/30851.9 = 4.296$ Ans. ***** *Efficiency of the machine:*

We know that the efficiency,

$$\eta = \frac{\tan \theta}{\tan (\theta + \phi)} = \frac{0.08}{\tan(4.574 + 7.4)} = 0.377 = 37.7 \% \text{ Ans.}$$

Problem 6

The mean diameter of the screw jack, having pitch of 10 mm, is 50 mm. A load of 20 kN is lifted through a distance of 170 mm. Find the work done in lifting the load and efficiency of the screw jack when

- (i) the load rotates with the screw, and
- (ii) the load rests on the loose head which does not rotate with the screw

The external and internal diameters of the bearing surface of the loose head are 60 mm and 10 mm respectively. The coefficient of friction for the screw and the bearing surface is 0.08.

Given:

$$p = 10 \text{ mm};$$
 $D_{ms} = 50 \text{ mm};$ $W = 20 \text{ kN};$ $D_{co} = 60 \text{ mm}$
 $D_{ci} = 10 \text{ mm}$ $\mu_s = \mu_c = \tan \phi = 0.08$

Solution:

For single start thread l = p

$$\tan \theta = \frac{p}{\pi D_{ms}} = \frac{10}{\pi \times 50} = 0.0637$$
$$\therefore \theta = \tan^{-1} 0.0637 = 3.645^{\circ}$$
$$\tan \phi = 0.08$$
$$\therefore \phi = \tan^{-1} 0.08 = 4.574^{\circ}$$

Torque required to lift the load,

$$T_R = W \tan(\phi + \theta) \frac{D_{ms}}{2} = 20000 \tan(4.574 + 3.645) \times \frac{50}{2} = 72220.36 \text{ N-mm}$$

Since the load is lifted through a vertical distance of 170 mm and the distance moved by the screw in one rotation is 10 mm (equal to pitch), therefore number of rotations made by the screw.

N = 170 / 10 = 171. When the load rotates with the screw. Work done in lifting the load = T × 2\piN = 72220.36 × 2 π × 17

Efficiency of the screw jack

$$\eta = \frac{\tan\theta}{\tan(\theta + \phi)} = \frac{0.0637}{\tan(3.645 + 4.574)} = 0.441 = 44.1\%$$
 Ans. *****

2. When the load does not rotate with the screw

We know that the mean diameter of the bearing surface, $D_{mc} = \frac{D_{co} + D_{ci}}{2} = \frac{60 + 10}{2} = 35 \text{ mm}$

And torque required to overcome friction at the screw and the collar

$$T = W \tan (\phi + \theta) \frac{D_{ms}}{2} + W \mu_c \frac{D_{mc}}{2}$$

= 20000 tan (4.574 + 3.645) $\frac{50}{2}$ + 20000 × 0.08 × $\frac{35}{2}$

T = 100220.36 N-mm = 100.22 N-m

Work done by the torque in lifting the load = $T \times 2\pi N = 100220.36 \times 2\pi \times 17$

We know that the torque required to lift the load, neglecting friction,

$$T_0 = W \tan \theta \frac{D_{ms}}{2} = 20000 \times 0.0637 \times \frac{50}{2} = 31850 \text{ N-mm} = 31.85 \text{ N-m}$$

Efficiency of the screw jack $\eta = T_o / T = 31.85 / 100.22 = 0.3178 = 31.78$ % Ans. *

Problem 7

The arrangement of a bench mounted hoisting system is shown in Figure 6.2. The hoist consists of a vertical post which supports a jib pivoted at B. The hoist is operated by hand via the tommy bar at H and the nut and screw arrangement. The screw is a double start square thread with an outside diameter of 40 mm, and a pitch of 15 mm, with a coefficient of friction of 0.1. The collar bearing has outside and inside diameters of 80 mm and 25 mm respectively and a coefficient of friction of 0.2. Determine

- (a) the distance X for the load W to be raised 120 mm for one turn of the nut
- (b) the maximum load W that can be lifted by the hoist when a load of 300 N is applied to the tommy bar at H
- (c) the maximum load W that can be lifted by the hoist if the collar bearing was replaced by rolling element bearing and for the same applied loading at H.



Figure 6.2

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D_o = 40 \text{ mm}, \quad p = 15 \text{ mm}, \quad n = 2, \quad D_{co} = 80 \text{ mm}, \quad D_{ci} = 25 \text{ mm}
\mu_s = \tan \phi = 0.1, \quad \mu_c = 0.2, \quad P = 300 \text{ N}, \quad L = 300 \text{ mm}
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Since the load which is placed at the end of jib is lifted through a vertical distance of 120 mm and the distance moved by the screw in one rotation is 30 mm (equal to pitch × number of start = 15×2 =30), therefore from the similar triangle BCC' and BDD',



Figure 6.3

$$\frac{BC}{CC'} = \frac{BD}{DD'}$$
$$\frac{X}{30} = \frac{720}{120}$$
$$\therefore X = \frac{720}{120} \times 30 = 180 \text{ mm} \quad \text{Ans.} \quad \text{*}$$

Let W = Maximum load that can be lifted by the hoist in Newton. Therefore, the load required to be lifted by the screw is:

$$W_s \times 180 = W \times 720$$
$$W_s = W \times \frac{720}{180}$$
$$= 4W$$

We know that mean diameter of the screw,

$$D_{ms} = D_o - \frac{p}{2} = 40 - \frac{15}{2} = 32.5 \, mm$$

Since the screw has double start, the lead of the screw is

$$l = n \times p = 2 \times 15 = 30 \ mm$$

$$\therefore \ \tan \theta = \frac{l}{\pi D_{ms}} = \frac{30}{\pi \times 32.5} = 0.294$$

$$\therefore \theta = \tan^{-1} 0.294 = 16.37^{\circ}$$

$$\tan \phi = 0.1$$

$$\therefore \phi = \tan^{-1} 0.1 = 5.71^{\circ}$$

We know that the mean diameter of the bearing surface, $D_{mc} = \frac{D_{co} + D_{ci}}{2} = \frac{80 + 25}{2} = 52.5 \text{ mm}$

Total torque required,

$$T = 4W \tan(\phi + \theta) \frac{D_{ms}}{2} + 4W \mu_c \frac{D_{mc}}{2}$$
$$= 4W \tan(5.71 + 16.37) \frac{32.5}{2} + 4W \times 0.2 \times \frac{52.5}{2}$$

T = 47.36 W N-mm

We also know that torque required at the end of a spanner,

 $T = P \times L = 300 \times 300 = 90000 \ N - mm$

Equating the two values of torque,

$$90000 = 47.36 W$$

 $W = 1900 N.$ Ans. *

(c) If the collar bearing is replaced by the rolling element bearing, then the friction is negligible. Total torque required to lift the load W is

$$T = 4W \tan(\phi + \theta) \frac{D_{ms}}{2}$$

= 4W \tan(5.71 + 16.37) $\frac{32.5}{2}$
T = 26.36 W N-mm

We also know that torque required at the end of a spanner,

 $T = P \times L = 300 \times 300 = 90000 \ N - mm$

Equating the two values of torques,

$$90000 = 26.36 W$$

 $W = 3414 N.$ Ans. *

Problem 8

The operating spindle C of the lifting jack shown in Figure 6.4 is threaded at both ends to engage the nuts A and B. Nut A has right-hand thread, and B has a left-hand thread. Both threads are square in cross section, each having a pitch of 6 mm and a major diameter of 30 mm. If the coefficient of friction for each nut is 0.12, and all the side links are inclined at 35° to the vertical. Determine

- (a) the axial load on the screw thread;
- (b) the torque required to raise a load of 10 kN;
- (c) the efficiency of the screw jack and whether the screw is self-reversing.



Figure 6.4

Given data:



Solution:

(a) First let us draw force equilibrium diagram of the arrangement to find out the axial load on the screw.



From above Figure 6.5

2 F cos 35° =10 kN F = $\frac{10}{2 \cos 35}$ = 6.1 kN

Axial load = F sin 35° = 6.1 sin 35° = 3.5 kN

Total axial load = $2 \times 3.5 = 7$ kN Ans. *****

(b) We know that mean diameter of the screw,

$$D_{ms} = D_o - \frac{p}{2} = 30 - \frac{6}{2} = 27 \, mm$$

Since the screw is single start, the lead of the screw is l = p

$$\therefore \tan \theta = \frac{l}{\pi D_{ms}} = \frac{6}{\pi \times 27} = 0.0707$$
$$\therefore \theta = \tan^{-1} 0.0707 = 4.046^{\circ}$$
$$\tan \phi = 0.12$$

 $\therefore \phi = \tan^{-1} 0.12 = 6.84^{\circ}$

Torque required to lift the load W by single nut is

T = W tan
$$(\phi + \theta) \frac{D_{ms}}{2}$$

= 7×tan $(6.84 + 4.046) \frac{27}{2}$

T = 18.17 kN-mm or N-m

Since the lifting jack has two nuts the total torque required to lift load is

 $= 2 \times 18.17 = 36.34$ kN-mm or N-m Ans. *****

(c) Efficiency of the screw jack

$$\eta = \frac{\tan\theta}{\tan(\theta + \phi)} = \frac{0.0707}{\tan(4.046 + 6.84)} = 0.3676 = 36.76\%$$

Since $\phi > \theta$, therefore the screw is **not self-reversing.** Ans. *****

Problem 9

A hand operated press for driving studs into wooden workpiece is shown in Figure 6.6. The press is operated by a hand wheel that drives a square threaded power screw that moves the press head up and down about the pivot as shown. The power screw thread has a major and minor diameter of 50 mm and 40 mm respectively and is double start. A plain bearing having an effective diameter of 90 mm in the arm takes the thrust from under the hand wheel. The coefficients of friction of the nut/screw and plain bearing are 0.13 and 0.27 respectively, and a friction torque of 15 N-m is present due to frictional resistance at the pivot. Determine

(a) the speed of the press head when driving the stud into the wooden workpiece when the hand wheel is rotated at 40 rev/min

(b) the driving force achieved at the press head when a torque of 18 N-m is applied to the hand wheel.



Figure 6.6

Given data:

$D_{mc} = 90 \text{ mm},$	n = 2,	$D_{so} = 50 \text{ mm},$	$D_{si} = 40 \text{ mm}$	$\mu_{\rm s} = \tan \phi = 0.13,$
$\mu_{c} = 0.27,$	T _{friction} =15 N-m,	T = 18 N-m = 18	8000 N-mm,	$\omega_H = 40 \text{ r.p.m}$

Mean diameter of the screw, $D_{ms} = \frac{D_{so} + D_{si}}{2} = \frac{50 + 40}{2} = 45 \text{ mm}$

We know that mean diameter of the screw,

$$D_{ms} = D_o - \frac{p}{2}$$
$$45 = 50 - \frac{p}{2}$$

 $\therefore p = 10 \text{ mm}$

Lead of the screw, $1 = n \times p = 2 \times 10 = 20 \text{ mm}$

(a) Speed of hand wheel, $\omega_H = 40 \text{ r.p.m} = 0.667 \text{ rev/sec}$

One revolution of the hand wheel moves the screw 20 mm (i.e, equal to the lead of the screw), therefore 0.667 rev/sec of hand wheel moves the screw by $20 \times 0.667 = 13.34$ mm/sec

Since the arm is pivoted at one end and the press head is placed 200 mm from both pivot and hand wheel, the proportionate speed of the press head is given by

$$13.34 \times \frac{200}{400} =$$
 6.67 mm/sec Ans. *****

(b) Since the frictional torque of 15 N-m is present at pivot, the proportional frictional force at the screw thread is

$$F_{\text{friction}} = \frac{15}{(200 + 200) \times 10^{-3}} = 37.5 \text{ N}$$
$$\tan \theta = \frac{l}{\pi D_{ms}} = \frac{20}{\pi \times 45} = 0.1415$$
$$\therefore \theta = \tan^{-1} 0.1415 = 8.05^{\circ}$$
$$\tan \phi = 0.13$$
$$\therefore \phi = \tan^{-1} 0.13 = 7.4^{\circ}$$

We know that torque applied at the hand wheel is given by,

$$T = W \tan\left(\phi + \theta\right) \frac{D_{ms}}{2} + W \mu_c \frac{D_{mc}}{2} = W \left[\tan\left(\phi + \theta\right) \frac{D_{ms}}{2} + \mu_c \frac{D_{mc}}{2} \right]$$

In the above expression the term W is the addition of driving force (F_{Drive}) at the handle and the frictional force at screw thread

$$\therefore T = \left(F_{\text{Drive}} + F_{\text{friction}}\right) \left[\tan\left(\phi + \theta\right) \frac{D_{\text{ms}}}{2} + \mu_c \frac{D_{\text{mc}}}{2} \right]$$

$$18000 = (F_{Drive} + 37.5) \left[tan (7.4 + 8.05) \frac{45}{2} + 0.27 \times \frac{90}{2} \right]$$

$$18000 = (F_{Drive} + 37.5) \ 16.64$$

$$F_{Drive} = 1044 \ N$$

The above driving force is obtained at the hand wheel. The proportional driving force obtained at the press head is

F =
$$1044 \times \frac{400}{200}$$
 = 2088 N Ans. *****