Glasgow Caledonian University

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Module: Engineering Design and Analysis 2

Epicyclic Gear and Power Screw Summary

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Original Epicyclic and Power Screw notes prepared by G.K. Vijayaraghavan (2006), Original Clutch, Belt and Brake Systems by Dr M. Macdonald, Updated notes prepared by A. Cowell (2017)

10A. Epicyclic Gear Analysis

The epicyclic gear trains may be *simple* or *compound*.

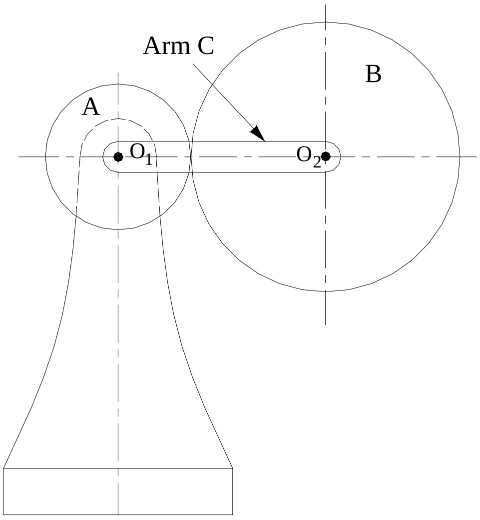

Figure 10A.1: Simple Gear Train

Velocity ratio of epicyclic gear train

Consider an epicyclic gear train as shown in Figure 5.1.

Let

$z_A \Rightarrow$ Number of teeth on gear A, and

$z_B \Rightarrow$ Number of teeth on gear B.

First of all, let us suppose that the arm is fixed. Therefore the axes of both the gears are also fixed relative to each other. The relationship between the rotational speed for the two gears is then represented by the equation:

$$\frac{N_B}{N_A} = \frac{z_A}{z_B} \qquad (10A.1)$$

Therefore, when the gear A makes one revolution anticlockwise ($N_A = 1$), the gear B will make $\frac{z_A}{z_B}$ revolutions clockwise (i.e. N_B). Assuming the anticlockwise rotation as positive and clockwise as negative, we may say that when gear A makes +1 revolution, then the gear B will make $\left(-\frac{z_A}{z_B}\right)$ revolutions. This relationship provides the basis for the development of the table of motions. This statement of relative motion is entered in the first row of the table. This table of motions will be used to solve all epicyclic gear problems in this course.

Secondly, if the gear A makes $+x$ revolutions, then the gear B will make $\left(-x \times \frac{z_A}{z_B}\right)$ revolutions. This statement is entered in the second row of the table. In other words, multiply the each motion (entered in the first row) by x.

Thirdly, each element of an epicyclic train is given $+y$ revolutions and entered in the third row. Finally, the motion of each element of the gear train is added up and entered in the fourth row.

Table of motions for Simple Gear Train

Step no	***Conditions of motion***	***Revolutions of element***		
		Arm C	***Gear A***	***Gear B***
1.	Arm fixed gear *A* rotates through +1 revolution i.e., 1 rev anticlockwise.	0	+ 1	$-\frac{z_A}{z_B}$
2.	Arm fixed gear *A* rotates through +*x* revolution.	0	$+x$	$-x\frac{z_A}{z_B}$
3.	Add +*y* revolutions to all elements.	$+y$	$+y$	$+y$
4.	Total motion.	$+y$	$x+y$	$y-x\frac{z_A}{z_B}$

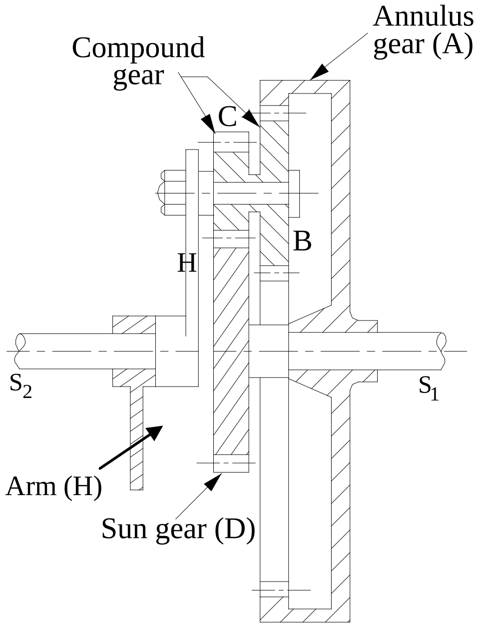

Figure 10A.2: Compound Gear Train

The annulus gear A meshes with gear B and the sun gear D meshes with the gear C. It may be noted that when the annulus gear is fixed, the sun gear provides the drive and when the sun gear is fixed, the annulus gear provides the drive. In both cases, the arm acts as follower.

Let z_A, z_B, z_C and z_D be the teeth and N_A, N_B, N_C and N_D be the speeds for the gears A, B, C, and D respectively. A little consideration will show that when the arm is fixed and the sun gear D is turned anticlockwise, then the compound gear B-C and the annulus gear A will rotate in the clockwise direction. Thus, from Equation (10A.1) with $N_D = 1$ (that is, gear D is turned one rotation):

$$N_C = \frac{z_D}{z_C} \tag{10A.2}$$

As B-C is a compound gear, $N_B = N_C$, and the next speed ratio to consider is N_A/N_B. Therefore;

$$\frac{N_A}{N_B} = \frac{z_B}{z_A} \tag{10A.3}$$

Re-arranging for N_A:

$$N_A = N_B \times \frac{z_B}{z_A} \tag{10A.4}$$

Substituting the value for N_B from Equation (10A.2) (equal to N_C): $N_A = \frac{z_D}{z_C} \times \frac{z_B}{z_A}$ (10A.5)

The motions of rotation of the various elements are then shown in the table below. The equations above can be used to generate the magnitude of the revolution value, but the directions must be taken from the following rules:

1. Gears that mesh external teeth to external teeth rotate in opposite directions.
2. Gears that mesh external teeth to internal teeth rotate in the same direction.

Thus, in the table below, compound gear B-C rotates in the opposite direction to gear D, and gear A rotates in the same direction as compound gear B-C.

Table of Motions for Compound Gear Train

Step no	***Conditions of motion***	***Revolutions of element***			
		Arm	***Gear D***	***Compound gear B-C***	***Gear A***
1.	Arm fixed gear D rotates through +1 revolution.	0	+ 1	$-\frac{z_D}{z_C}$	$-\frac{z_D}{z_C}\times\frac{z_B}{z_A}$
2.	Arm fixed gear D rotates through +*x* revolution.	0	$+x$	$-x\frac{z_D}{z_C}$	$-x\frac{z_D}{z_C}\times\frac{z_B}{z_A}$
3.	Add +y revolutions to all elements.	$+y$	$+y$	$+y$	$+y$
4.	Total motion.	$+y$	$x+y$	$y-x\frac{z_D}{z_C}$	$y-x\frac{z_D}{z_C}\times\frac{z_B}{z_A}$

Nomenclature for Epicyclic Gear Trains

Symbol	**Description**	**Unit**
d	Gear diameter	m
N	Rotational speed	rpm
x	Notional number of rotations of gear (normally planet gear)	rpm
y	Notional number of rotations of arm	rpm
z	Number of teeth	-

10B. Power Screw Analysis

The following terms are important for the study of screws.

1. ***Helix:*** It is the curve traced by a particle while describing a circular path at a uniform speed and advancing in the axial direction at a uniform rate. In other words, it is the curve traced by a particle while moving along a screw thread.
2. ***Pitch:*** It is the distance from a point of a screw to a corresponding point on the next thread measured parallel to the axis of the screw.
3. ***Lead:*** It is the distance a screw thread advances axially in one turn.
4. ***Depth of thread:*** It is the distance between the top and bottom surfaces of a thread (also known as crest and root of a thread).
5. ***Single-threaded screw***: If the lead of a screw is equal to its pitch, it is known as single threaded screw.
6. ***Multi-threaded screw:*** If more than one thread is cut in one lead distance of a screw, it is known as multi-threaded screw e.g. in a double threaded screw, two threads are cut in one length. In such cases, all the threads run independently along the length of the rod. Mathematically,

   Lead = Pitch × Number of threads
7. ***Slope of thread(θ):*** It is the inclination of the thread with the horizontal. Mathematically,

$$\tan\theta = \frac{Lead\ of\ screw}{Circumference\ of\ screw} = \frac{l}{\pi D_{ms}} \qquad (10B.1)$$

l = lead of the screw = $n\,p$
p = pitch of the screw
D_{ms} = Mean diameter of the screw, and
n = number of start of threads

Screw Jack

The screw jack is a device for lifting heavy loads by applying a comparatively smaller effort at its handle. The principle on which a screw jack works is similar to that of an inclined plane.

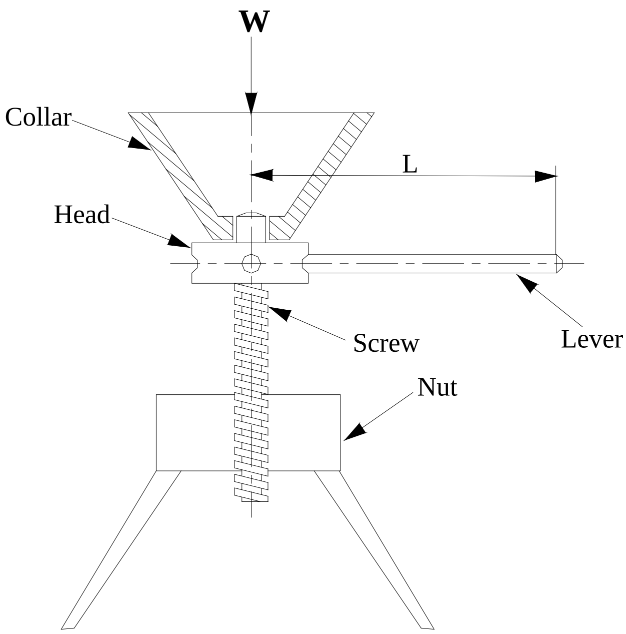

Figure 10B.1 Screw jack with thrust collar

Let p = pitch of the screw

D_{ms} = Mean diameter of the screw

D_{mc} = Mean diameter of the collar

θ = helix angle

W = load to be lifted

μ_s = coefficient of friction of screw thread, i.e., between the screw and nut

= tan ϕ, where ϕ is the friction angle

μ_c = coefficient of friction of collar bearing

Torque required to lift the weight is given by the equation

$$T = W \tan(\phi + \theta)\frac{D_{ms}}{2} + W\mu_c\frac{D_{mc}}{2} \qquad (10B.2)$$

When there is no collar bearing then the above equation is reduced to

$$T = W \tan(\phi + \theta)\frac{D_{ms}}{2} \qquad (10B.3)$$

If an effort P is applied at the end of a lever of arm length L, then the total torque required to overcome friction must be equal to the torque applied at the end of the lever.

$$T = W \tan(\phi + \theta)\frac{D_{ms}}{2} + W\mu_c\frac{D_{mc}}{2} = P \times L \qquad (10B.4)$$

The nominal diameter of a screw thread is also known as outside diameter or major diameter. The core diameter of a screw thread is also known as inner diameter or root diameter or minor diameter.

When the nominal diameter (D_o) and/or the core diameter (D_c) of the screw thread is given, the mean diameter of the screw is given by:

$$D_{ms} = \frac{D_O + D_C}{2} = D_O - \frac{p}{2} = D_C + \frac{p}{2} \qquad (10B.5)$$

We know that, the effort required to lift the load (W) when friction is taken into account,

$$T = W \tan(\phi + \theta)\frac{D_{ms}}{2} + W\mu_c\frac{D_{mc}}{2} \qquad (10B.6)$$

θ = Helix angle

ϕ = Angle of friction, and

μ = Coefficient of friction between the screw and nut = tan ϕ = 0

If there is no friction between the screw and the nut, then ϕ will be equal to zero, thus the value of torque T_0 necessary to raise the load will then be given by the equation:

$$T_0 = W \tan\theta \frac{D_{ms}}{2} \qquad (10B.7)$$

Efficiency, $$\eta = \frac{\textit{Torque required to move the load, neglecting friction}}{\textit{Torque required to move the load, including screw and collar friction}}$$

$$= \frac{T_0}{T} = \frac{W \tan\theta \frac{D_{ms}}{2}}{W \tan(\phi + \theta)\frac{D_{ms}}{2} + W\mu_c\frac{D_{mc}}{2}} \qquad (10B.8)$$

If there is no thrust bearing present in the arrangement, then the above efficiency term is reduced to

$$\text{Efficiency, } \eta = \frac{\tan\theta}{\tan(\theta+\phi)} \tag{10B.9}$$

The torque required to lower the load is

$$T = W \tan(\phi - \theta)\frac{D_{ms}}{2} + W\mu_c\frac{D_{mc}}{2} \tag{10B.10}$$

In the above expression if ϕ < θ the torque required to lower the load may be negative. In other words, the load will start moving downwards without the application of any torque. This, of course, depends on the magnitude of the second term. For this condition the torque required to lower the load should be checked to ensure it is positive. If this torque is negative, such a condition is known as overhauling of screws. If however, ϕ > θ , the torque required to lower the load will always be positive, indicating that an effort is applied to lower the load. Such a screw is known as a self-locking screw. In other words a screw will be always be self-locking if the friction angle is greater than helix angle or the coefficient of friction is greater than the tangent of the helix angle, i.e μ or tan ϕ > tan θ.

Nomenclature for Power Screws

Symbol	**Description**	**Unit**
D_{mc}	Mean diameter of the collar	m
D_{ms}	Mean diameter of the screw	m
D_c	Core (root) diameter of the screw	m
D_o	Nominal (outside) diameter of the screw	m
l	Lead distance (axial movement for one rotation)	m
L	Length of lever arm	m
n	Number of threads	-
p	Screw pitch	m
P	Lever arm load	N
T	Screw torque	Nm
T_0	Ideal screw torque (no friction)	Nm
W	Load to be lifted	N
η	Efficiency	-
ϕ	Screw thread friction angle	degrees
μ_χ	Coefficient of friction of collar bearing	-
μ_σ	Coefficient of friction of screw thread	-
θ	Angle of thread/helix angle	degrees