Bending Stress

Worked Example No.3

A beam ABCD is simply-supported at A and D and is loaded at B and C as shown in Figure Ex.3(a). The beam has the cross-section shown in Figure Ex.3(b).

(a) Determine the magnitude of the force reactions at supports A and D.

(b) Sketch the shear force and bending moment diagrams for the beam, indicating all significant values.

(c) State the magnitude and position of the maximum bending moment acting on the beam.

(d) Determine the second moment of area about the neutral axis (NA) for the beam cross-section.

(e) Determine the maximum bending stress set-up in the beam material.

(f) Sketch the bending stress distribution across the beam cross-section at the point of maximum bending stress.

(g) If the beam is of cast iron and has an allowable stress in bending of 25 MN/m², determine the factor of safety for the beam.

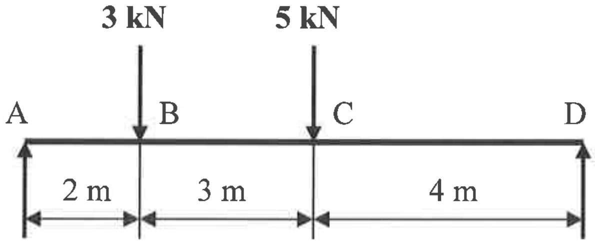


Figure Ex.3(a): Loaded Beam

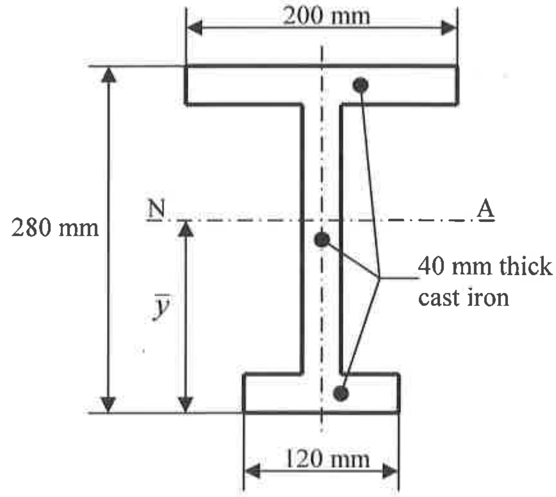


Figure Ex.3(b): Beam Cross-Section

Solution

a) Reactions R_A & R_D :

$$\Sigma M_A \overset{+}{\curvearrowright} = 0:$$

$$(3\times 2)+(5\times 5)-9R_D = 0$$

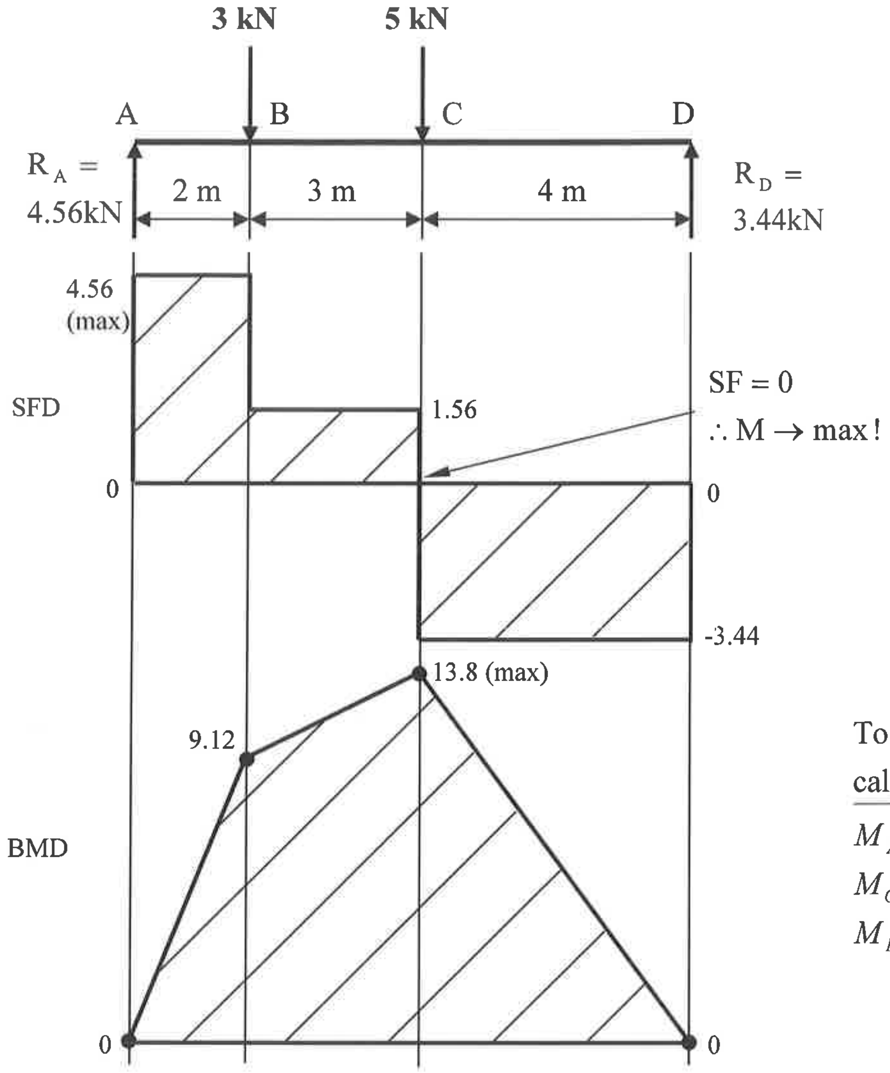
$$\therefore\ R_D = 3.44kN$$

$$\Sigma F_Y = 0(+\uparrow):$$

$$R_A - 3 - 5 + 3.44 = 0$$

$$\therefore R_A = 4.56kN$$

b) SF and BM Diagrams :



To calculate BMs, use SFD areas or, by calculating at each point on beam :

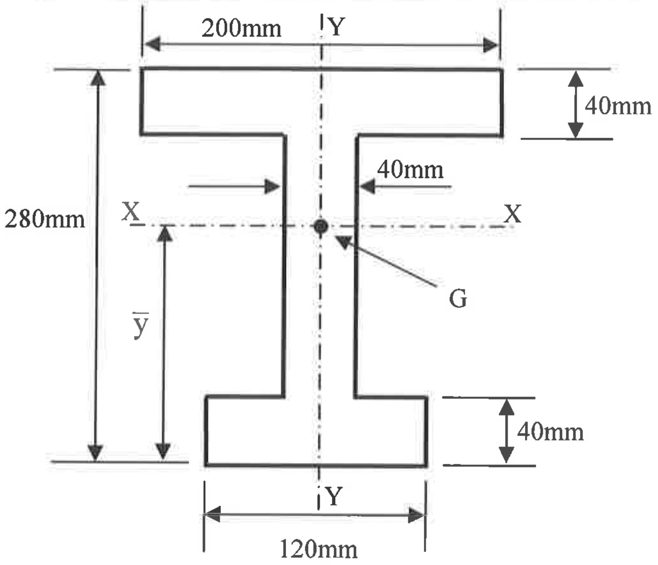
$M_A = 0$; $M_B = (4.56 \times 2) = 9.12kNm$

$M_C = (4.56 \times 5) - (3 \times 3) = 13.8kNm$

$M_D = 0$

c) $M_{MAX} = 13.8$kNm acting at C.

d) 2nd Moment of Area I :



Position of Centroid G :

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(120\times 40)20 + (200\times 40)140 + (200\times 40)260}{(120\times 40) + (200\times 40) + (200\times 40)}$$

$$\therefore \bar{y} = 158.5mm$$

To calculate I - use Parallel Axis Theorem :

$$I_{XX} = \Sigma\left(I_G + Ah^2\right) = I_1 + I_2 + I_3$$

$$= \left[\frac{120\times 40^3}{12} + (120\times 40)(138.5)^2\right] + \left[\frac{40\times 200^3}{12} + (40\times 200)(18.5)^2\right] + \left[\frac{200\times 40^3}{12} + (200\times 40)(101.5)^2\right]$$

$$\therefore I_{XX} = 205.6\times 10^6\, mm^4 \text{ or } 205.6\times 10^{-6}\, m^4$$

e) Max. Bending Stress σ:

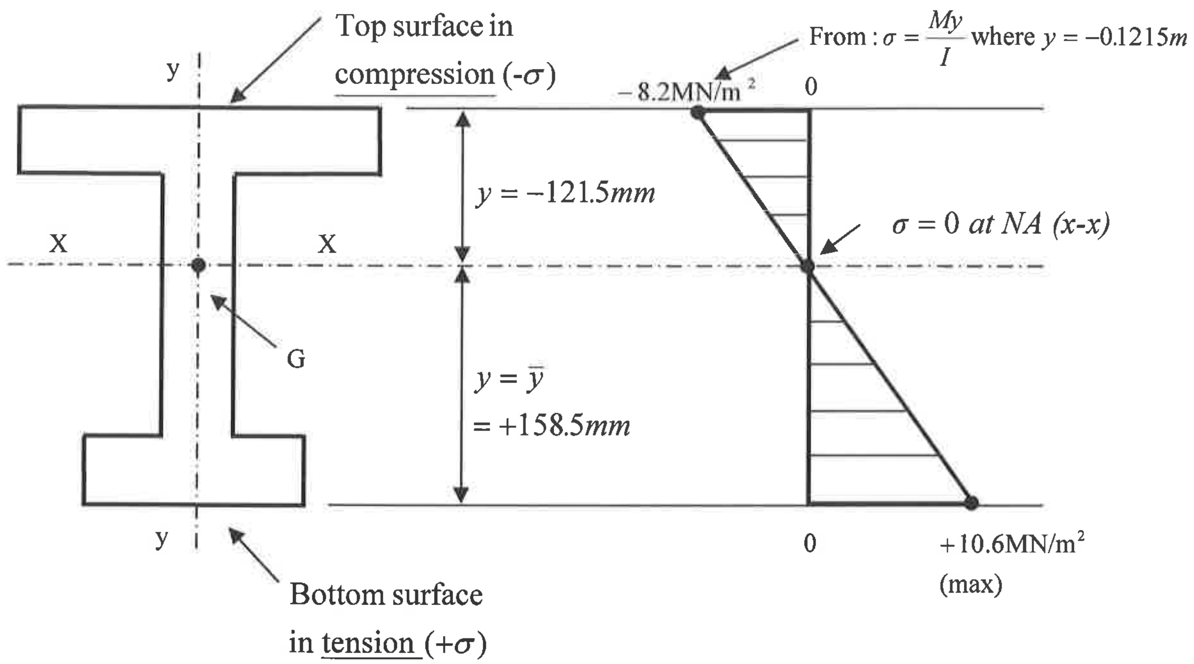
Theory of Bending: $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

where $M = M_{max} = 13.8$ kNm $= 13.8 \times 10^3$ Nm
$I = 205.6 \times 10^{-6}\ m^4$
$y = \bar{y} = 158.5$ mm $= 0.1585$ m

$$\therefore \quad \sigma = \frac{My}{I} = \frac{13.8x10^3 x0.1585}{205.6x10^{-6}} = 10.6 \times 10^6\ N/m^2$$

$\therefore$ $\sigma = 10.6\ MN/m^2$

f) Bending Stress Distribution:



g) Factor of Safety FoS:

$$FoS = \frac{\text{Max. or Allowable Stress}}{\text{Actual Stress}}$$

$$= \frac{25}{10.6}$$

$$\therefore FoS = 2.36$$