Worked Example 1

A short hollow cast-iron column shown in Figure Ex. 1 is to support a vertical load of 1 MN. The external diameter of the column is 250 mm and the thickness is 25 mm. Find the maximum allowable eccentricity of this load if the maximum tensile stress is not to exceed 30 N/mm^2. What is then the value of the maximum compressive stress?

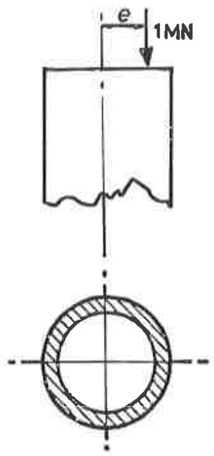

Figure Ex. 1

Solution
(working throughout in N and mm; 1 MN = 10^6 N).

$$A = \frac{\pi}{4}\left(250^2 - 200^2\right) = 17.7 \text{ x } 10^3 \text{mm}^2$$

$$I = \frac{\pi}{64}\left(250^4 - 200^4\right) = 113.5 \text{ x } 10^6 \text{mm}^4$$

direct stress $\sigma_d = - \dfrac{1 \text{ x } 10^6}{17.7 \text{ x } 10^3} = -56.5 \text{ N/mm}^2 \text{(compressive)}$

bending moment M = $10^6 e$, where e = eccentricity of the load from the centroid in mm.

maximum bending stress $\sigma_b = \dfrac{My}{I} = \pm \dfrac{10^6 \text{ x } e \text{ x } 125}{113.5 \text{ x } 10^6} = \pm\, 1.1e$ N/mm^2

maximum tensile stress $\sigma_{t\,max} = 1.1\, e - 56.5 = 30$ N/mm^2 $\quad\therefore\quad$ e = **78.5 mm**

maximum compressive stress $\sigma_{c\,max} = -\ 1.1\ e - 56.5 = -\ (1.1 \text{ x } 78.5) - 56.5$

$= 143$ N/mm^2 = **143 MN/m^2**

Worked Example 2

A screw clamp is tightened on a proving ring as shown in Figure Ex. 2. From measurement of the deflection of the ring, the clamping force is estimated as 11 kN. Find the maximum tensile and compressive stresses in the material at section A-B due to bending and direct loading.
Area of section = 480 mm^2, I_{xx} = 6.4 x 10^4 mm^4.

Figure Ex. 2

Solution

The resisting force exerted by the ring puts section A-B of the clamp under a direct tensile load of 11 kN and a bending moment = 11 x 10^3(0.09 + 0.016) = 1166 Nm about the axis X-X, through the centroid of section. Since the effect of the resisting force exerted by the ring is to tend to open out the clamp, the top portion of the section is in tension and the bottom in compression.

$$\text{Direct stress } \sigma_d = \frac{11 \times 10^3}{480 \times 10^{-6}} = 22.9 \times 10^6 \text{ N/m}^2 = 22.9 \text{ MN/m}^2 \text{ (tension)}$$

$$\text{Bending stress at top face } \sigma_b = \frac{M}{I} y = \frac{1166}{6.4 \times 10^4 \times 10^{-12}} \times 0.016 = 291.5 \times 10^6 \text{ N/m}^2$$

$$= 291.5 \text{ MN/m}^2 \text{ (tension)}$$

$$\text{Bending stress at bottom face } \sigma_b = -\frac{1166}{6.4 \times 10^4 \times 10^{-12}} \times 0.024 = -437 \times 10^6 \text{ N/m}^2$$

$$= -437.3 \text{ MN/m}^2 \text{ (compression)}$$

∴ at top face, maximum stress = 22.9 + 291.5 = **314.4 MN/m^2** (tension)

∴ at bottom face, maximum stress = 22.9 – 437.3 = **- 414.4 MN/m^2** (compression)

Worked Example 3

A uniform masonry chimney of outside diameter 3.m, inside diameter 2.4 m is subject to a horizontal wind of load 1.8 kN/m of height as shown in Figure Ex. 3. The weight of masonry is 17.5 kN/m^3. Calculate the maximum chimney height to avoid tensile stress at the base section.

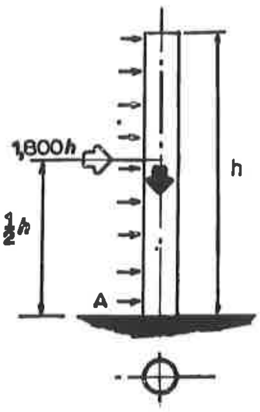

Figure Ex.3

Solution

Total wind load on height h (m), F = 1,800h (N)

Bending moment at base section due to this load, M = 1,800 h x ½ h = 900 h^2 (Nm)

$$I = \frac{\pi}{64}\left(3^4 - 2.4^4\right) = 2.35\ \text{m}^4$$

y_{max} = 1.5m

Maximum tensile stress at base $\sigma_b = \frac{My}{I} = \frac{900h^2 \times 1.5}{2.35}$ = 575 h^2 (N/m^2) at point A

Total weight of chimney = volume x specific weight = area of section x height x specific weight

= Ah x 17500 (N)

Compressive stress at base due to dead load $\sigma_d = -\frac{Ah \times 17500}{A} = -17500\,h$ (N/m^2)

For no tensile stress at base, total stress at A = 0. Therefore,

$575\,h^2 - 17{,}500\,h = 0$ ∴ h = **30.4 m**