Torsion of Shafts

Worked Example 1

(a) A solid shaft, 100 mm diameter, transmits 75 kW of power at a speed of 150 rev/min. Determine the value of the maximum shear stress set up in the shaft and the angle of twist per metre length of shaft if G = 80 GN/m^2.

(b) If the shaft were now bored in order to reduce weight to produce a tube of 100 mm outer diameter and 60mm inner diameter, what torque could be carried by the shaft if the same maximum shear stress is not to be exceeded?

Solution

(a) Solid shaft

d = 100 mm = 0.1 m

r = 0.05 m

P = 75 kW = 75x10^3 W

N = 150 rev/min

G = 80 GN/m^2 = 80x10^9 N/m^2

$$P = \frac{2\pi NT}{60} \quad \therefore \quad T = \frac{60P}{2\pi N} = \frac{60x(75x10^3)}{2\pi x150} = 4.77x10^3\,Nm$$

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L} \quad \therefore \quad \tau = \frac{Tr}{J} \text{ where } J = \frac{\pi d^4}{32} = \frac{\pi x0.1^4}{32} = 9.82x10^{-6}\,m^4$$

$$\therefore \quad \tau = \frac{Tr}{J} = \frac{(4.77x10^3)x0.05}{9.82x10^{-6}} = \boxed{24.3MN/m^2}$$

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L} \therefore \quad \theta = \frac{TL}{GJ} = \frac{(4.77x10^3)x1}{(80x10^9)x(9.82x10^{-6})} = 6.07x10^{-3}\,rad/m = \boxed{0.348^o/m}$$

(b) Modified shaft - hollow

D = 100 mm = 0.1 m

r = 0.05 m

d = 60 mm = 0.06 m – shaft drilled through to reduce weight!

P = 75 kW = 75x10^3 W

N = 150 rev/min

G = 80 GN/m^2 = 80x10^9 N/m^2

τ = 24.3 MN/m^2= 24.3x10^6 N/m^2

For hollow shaft, $J = \frac{\pi}{32}\left(D^4 - d^4\right) = \frac{\pi}{32}(0.1^4 - 0.06^4) = 8.545x10^{-6}\,m^4$

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L} \quad \therefore \quad T = \frac{\tau J}{r} = \frac{(24.3x10^6)x(8.545x10^{-6})}{0.05} = \boxed{4.15kNm}$$

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Worked Example 2

Determine the dimensions of a hollow shaft with a diameter ratio of 3:4, which is to transmit a power of 60 kW at a speed of 200 rev/min. The maximum shear stress in the shaft is limited to 70 MN/m^2 and the angle of twist is limited to 3.8° over a length of 4 m.
For the shaft material G = 80 GN/m^2.

Solution

D : d = 3 : 4
P = 60 kW = $60x10^3$ W
N = 200 rev/min
$\tau_{max} = 70\ MN/m^2 = 70x10^6\ N/m^2$
$G = 80\ GN/m^2 = 80x10^9\ N/m^2$
$\theta_{max} = 3.8^o$
L = 4 m

Max. shear stress condition:

$$P = \frac{2\pi NT}{60} \quad \therefore \quad T = \frac{60P}{2\pi N} = \frac{60x(60x10^3)}{2\pi x200} = 2.86x10^3\ Nm$$

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L} \quad \therefore \quad J = \frac{Tr}{\tau} \text{ where } J = \frac{\pi}{64}\left(D^4 - d^4\right), \text{ r = D/2 and d = 0.75D}$$

$$\therefore \quad J = \frac{\pi}{32}\left(D^4 - d^4\right) = \frac{\pi}{32}\left(D^4 - (0.75D)^4\right) = \frac{(2.86x10^3)x(D/2)}{70x10^6} = 20.43x10^{-6} D$$

$$\therefore \quad D^4(1-0.75^4) = \frac{(20.43x10^{-6})xDx32}{\pi} \quad \therefore \quad D^3 = 304.4 \text{ x } 10^{-6}$$

$$\therefore \quad D = 0.0673 \text{ m} = 67.3 \text{ mm, and, } \quad d = 0.75 \text{ x } 67.3 = 50.5 \text{ mm}$$

Max. angle of twist condition:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L} \quad \therefore \quad J = \frac{TL}{G\theta} = J = \frac{\pi}{32}\left(D^4 - d^4\right) = \frac{\pi}{32}\left(D^4 - (0.75D)^4\right) = \frac{(2.86x10^3)x4}{80x10^9 x3.8x(\pi/180)}$$

$$\therefore \quad D^4(1-0.75^4) = 21.96x10^{-6} \quad \therefore \quad D^4 = 32.12 \text{ x } 10^{-6}$$

$$\therefore \quad D = 0.0753 \text{ m} = 75.3 \text{ mm, and, } \quad d = 0.75 \text{ x } 75.3 = 56.5 \text{ mm}$$

Now, since both conditions require to be satisfied, the selected shaft sizes are:

$$\boxed{D = 75.3 \text{ mm, and, d} = 56.5 \text{ mm}}$$

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Worked Example 3

A power transmission shaft ABCD has the geometry shown in Figure Ex.3. If torques are applied to points A, B, C and D as shown, and the modulus of rigidity for the shaft material is 80 MN/m^2, using the data given:

i) draw the torque diagram for the shaft;
ii) determine the value and the position of the maximum shear stress;
iii) evaluate the total angle of twist of the shaft.

Data: $d_{AB} = 100$ mm, $d_{BC} = 150$ mm, $d_{CD} = 120$ mm
$L_{AB} = 1$ m, $L_{BC} = 2$ m, $L_{CD} = 2$ m
$T_A = 1$ kNm, $T_B = 12$ kNm, $T_C = 8$ kNm, $T_D = 3$ kNm

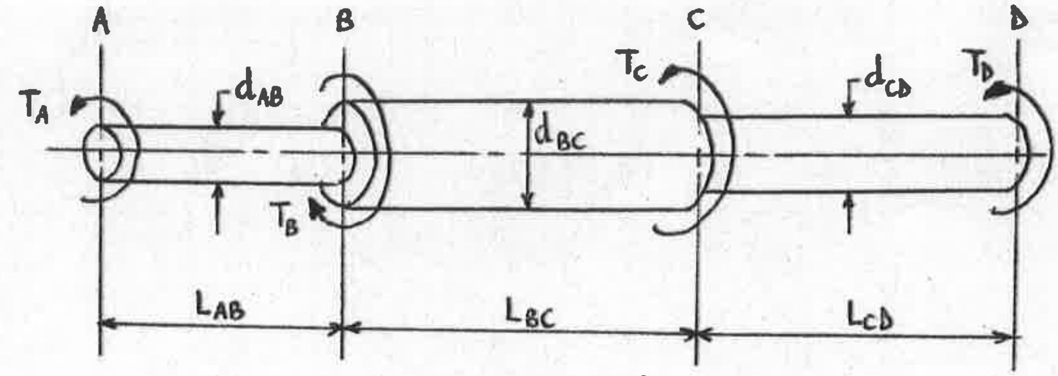
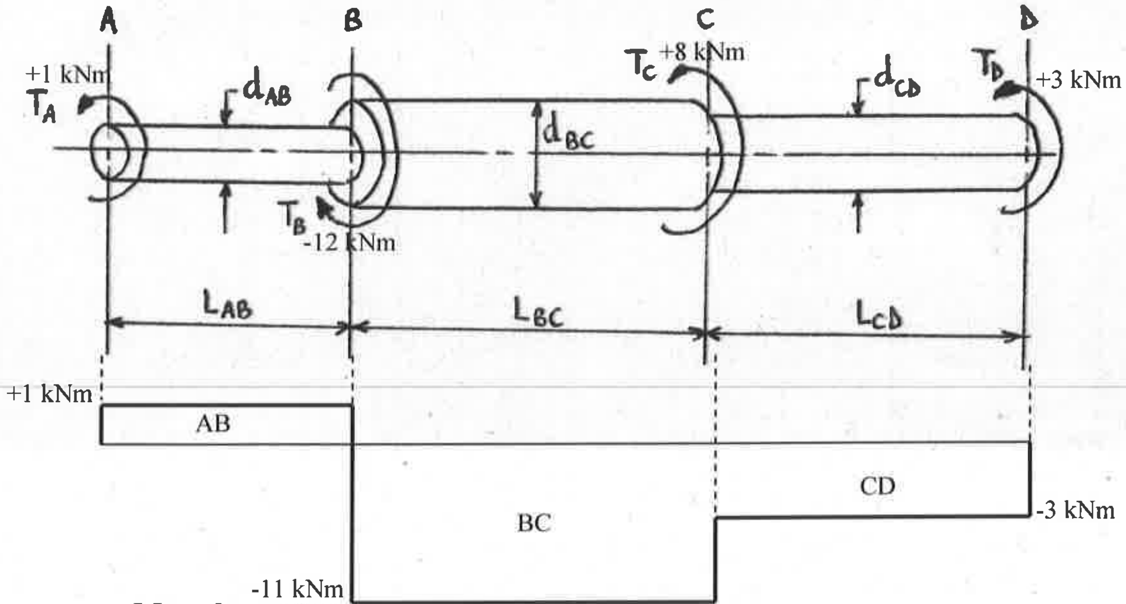


Figure Ex.3

Solution

i) **Torque Diagram:**



ii) **Max shear stress:**

$d_{AB} = 0.1$ m $\therefore$ $r_{AB} = 0.05$ m
$d_{BC} = 0.15$ m $\therefore$ $r_{BC} = 0.075$ m

$d_{CD} = 0.12$ m $\quad\therefore\quad r_{CD} = 0.06$ m

$T_{AB} = 1x10^3$ Nm

$T_{BC} = 11x10^3$ Nm

$T_{CD} = 3x10^3$ Nm

$$\therefore \quad J_{AB} = \frac{\pi}{32} D_{AB}{}^4 = \frac{\pi}{32} x0.1^4 = 9.82x10^{-6} m^4$$

$$\therefore \quad J_{BC} = \frac{\pi}{32} D_{BC}{}^4 = \frac{\pi}{32} x0.15^4 = 49.7x10^{-6} m^4$$

$$\therefore \quad J_{CD} = \frac{\pi}{32} D_{CD}{}^4 = \frac{\pi}{32} x0.12^4 = 20.36x10^{-6} m^4$$

Now, theory of torsion: $\dfrac{T}{J} = \dfrac{\tau}{r} = \dfrac{G\theta}{L} \quad \therefore \quad \tau_{AB} = \dfrac{T_{AB} r_{AB}}{J_{AB}} = \dfrac{(1x10^3)x0.05}{9.82x10^{-6}} = 5.1 MN/m^2$

$$\therefore \quad \tau_{BC} = \frac{T_{BC} r_{BC}}{J_{BC}} = \frac{(11x10^3)x0.075}{49.7x10^{-6}} = 16.6 MN/m^2$$

$$\therefore \quad \tau_{CD} = \frac{T_{CD} r_{CD}}{J_{CD}} = \frac{(3x10^3)x0.06}{20.36x10^{-6}} = 8.8 MN/m^2$$

Hence the max shear stress is 16.6 MN/m^2 acting on shaft portion BC.

iii) Total angle of twist:

Theory of torsion: $\dfrac{T}{J} = \dfrac{\tau}{r} = \dfrac{G\theta}{L} \quad \therefore \quad \theta = \dfrac{TL}{GJ}$ and $\theta_{AD} = \theta_{AB} + \theta_{BC} + \theta_{CD}$

$$\therefore \quad \theta_{AD} = \frac{(1x10^3)x1}{(80x10^9)x(9.82x10^{-6})} - \frac{(11x10^3)x2}{(80x10^9)x(49.7x10^{-6})} - \frac{(3x10^3)x2}{(80x10^9)x(20.36x10^{-6})}$$

$$= 0.00127 - 0.00553 - 0.00368$$

θ_{AD} = **-0.00794 rad = -0.455°** (*Note:* -ve indicates ACW direction of twist!)