School of Engineering & Built Environment

MEng/BEng(Hons) in:

Mechanical-Electronic Systems Engineering
Mechanical & Power Plant Systems
Electrical Power Engineering
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Module: Engineering Design & Analysis 2
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2D Strain Analysis: A Summary

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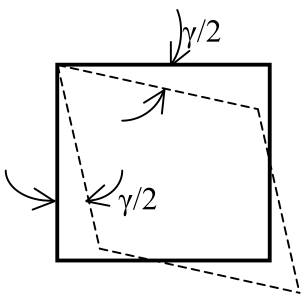
ENGINEERING DESIGN & ANALYSIS 2 (M2H721926)

2D Strain Analysis

Normal strain or direct strain is the change in length (dimension)/original length (dimension) and is denoted by ε (+ tensile, - compression).
Shear strain is a measure of the change of shape and is taken as the total angular change in any right angle, and is denoted γ, taken as positive corresponding to +ve shear stress and vice versa.



Also, $\dfrac{\tau_{xy}}{\gamma_{xy}}$ = constant = G (modulus of rigidity, N/m^2)

Also, recall Hooke's law: $E = \dfrac{\sigma}{\varepsilon}$ (E = modulus of elasticity, N/m^2)

And, E = 2G(1 + ν)

The strains at a point in a 2D stress system can be analysed in a similar manner to that for stresses (but more complex), and the following expressions apply:

$$\varepsilon_\theta = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta\,; \qquad \left(\frac{\gamma}{2}\right)_\theta = \frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta - \frac{\gamma_{xy}}{2}\cos 2\theta$$

Plotting values of ε_θ and $\left(\dfrac{\gamma}{2}\right)_\theta$ for varying values of θ gives the Mohr Circle of Strain, similar to the Mohr Circle of Stress.

The radius of the circle = $\sqrt{\left(\dfrac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\dfrac{\gamma_{xy}}{2}\right)^2}$

The centre of the circle lies at a distance from the origin = $\dfrac{\varepsilon_x + \varepsilon_y}{2}$ similar to the stress circle, and ε_1, ε_2 (Principal strains) are:

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}\,; \qquad \tan 2\beta = \frac{\gamma/2}{\dfrac{\varepsilon_x - \varepsilon_y}{2}}$$

The plane of maximum shear strain is at 45° to the principal plane.

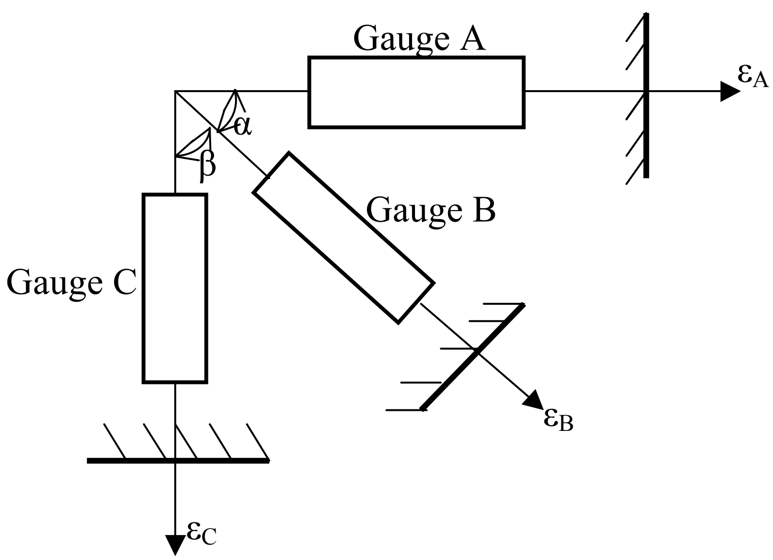
Strain Measurement
Strains at a point in a loaded component cannot be measured directly, but their effect in the form of direct strains can be measured by means of strain gauges or extensometers and stress calculations from these measurements.

The aim of the strain measurement is to find the principal stresses (and generally the major value) and their directions relative to some given reference direction. If the principal stress directions are known, a pair of strain gauges in these directions at a point will give the principal strains from which the principal stresses can be found by the usual stress-strain equations in two-dimensions.

If however the principal stress directions are unknown, a pair of strain gauges arranged at a known angle to each other at a point, do not give sufficient information to enable a Mohr Circle of Strain to be constructed for the strains at a point. The normal strains are known from the gauge readings but as the associated shear strains cannot be obtained from the readings, the Mohr Circle cannot be constructed.

Thus when principal stress directions are unknown, three strain gauges arranged at known angles to each other at a point (e.g. a strain rosette) will give the minimum data necessary to construct a Mohr Circle for the strain system at the point.

Assume the typical strain gauge rosette shown gives strain readings ε_A, ε_B, and ε_C – thus strains ε_A, ε_B, and ε_C, and angles α and β would all be known.

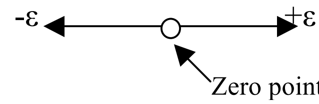


A typical strain gauge rosette arrangement.

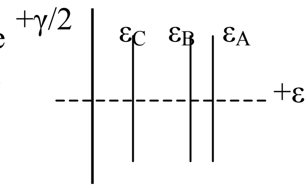
Mohr Circle of Strain – Method of Construction

Construction of the Mohr Circle for strains in 2D at a point on a loaded component would be as follows:

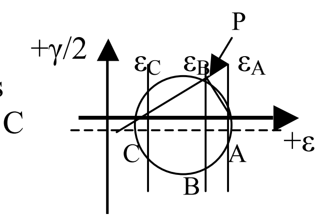
1. Draw any horizontal axis along which to measure normal or direct strains (ε). All normal strains (σ) are measured along this x-axis. Note that this <u>may or may not be the final horizontal axis for the Mohr Circle</u>. Choose arbitrarily, a zero strain point with $+\varepsilon$ to the right of this point and $-\varepsilon$ to the left.



2. From the arbitrarily chosen zero strain point, scale along the axis the given normal strains and draw a vertical line through the points, and through the zero point.



3. Choose any <u>point</u> P on the middle vertical line which corresponds to ε_B. From this point measure angle α towards ε_A and β towards ε_C, drawing lines to cut the verticals through ε_A and ε_C in A and C respectively. Points A and C are the points of the Mohr Circle of Strain which can then be drawn.



4. Having drawn the Mohr Circle and found its centre, draw in the correct normal strain axis ε through the centre of the circle. If point P is assumed to be the <u>Pole Point</u> on the Mohr Circle for plane B, plane B can be drawn vertically as shown and is then the plane of reference, hence PA and PC making α and β respectively with plane B at the pole point, cut the Mohr Circle in A and C respectively so that the points A and C represent the planes A and C, and the normal and shear strains on these planes. Note: the relative shear strains on planes A, B and C only emerge as a result of the Mohr Circle.

The principal strains are ε_1 and ε_2, the directions being at angle θ to the principal plane B by use of the pole point as shown.

Having found the principal strains ε_1 and ε_2, the principal stresses σ_1 and σ_2 are obtained as follows:

$$\sigma_1 = \frac{E}{1-\nu^2}\left[\varepsilon_1 + \nu\varepsilon_2\right]$$

$$\sigma_2 = \frac{E}{1-\nu^2}\left[\varepsilon_2 + \nu\varepsilon_1\right]$$

where E = Young's modulus of elasticity (N/m^2) and ν = Poisson's ratio for the component material.