Worked Example 2D Strain Analysis

A thin cylinder of internal diameter 300 mm is manufactured from 6 mm thick steel plate. The steel has a Young's modulus of 208 GN/m² and a Poisson's ratio of 0.28. The cylinder is subjected to an internal pressure 'p' and the 60° delta strain gauge rosette attached to the cylinder surface gave the strain readings as shown in Figure Ex.1. Determine:

- (a) the principal strains in the cylinder wall;
- (b) the longitudinal stress developed in the cylinder wall;
- (c) the magnitude of the internal pressure.

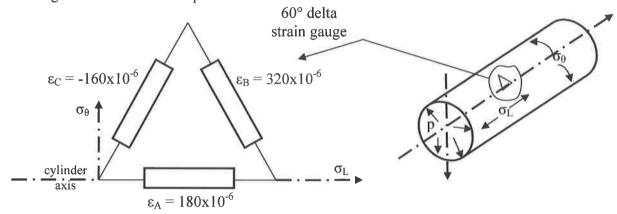
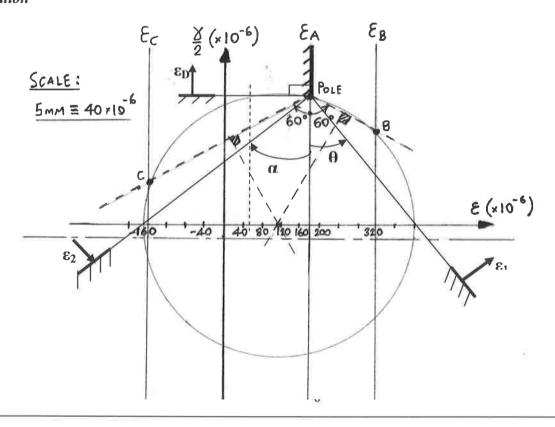


Figure Ex.1

Solution



Principal strains: (a)

$$\epsilon_1$$
 = 400 x 10 $^{\text{-}6}$ at θ = 38 $^{\text{o}}$ to gauge A

$$\epsilon_2 = -168 \times 10^{-6}$$
 at $\alpha = 52^{\circ}$ to gauge A

Longitudinal Stress: (b)

From the Mohr Circle of Strain:

$$\varepsilon_{\rm D} = \varepsilon_{\rm \theta} = 56 \text{ x } 10^{-6} \text{ at } 90^{\circ} \text{ to gauge A (as shown)} \quad \varepsilon_{\rm D} = \varepsilon_{\rm \theta}$$

Also, as given:

$$\varepsilon_{\rm A} = 180 \text{ x } 10^{-6} \ (= \varepsilon_{\rm L})$$

Longitudinal stress, $\sigma_L = \frac{E}{1 - v^2} [\varepsilon_A + v \varepsilon_D]$ (= axial stress!)

$$= \frac{208x10^9}{1 - 0.28^2} \left[(180x10^{-6}) + 0.28(56x10^{-6}) \right]$$

$$= 44.16 \text{ MN/m}^2$$

(c) **Internal Pressure:**

Longitudinal stress, $\sigma_L = \frac{pd}{4t}$

$$p = \frac{\sigma_L x 4t}{d} = \frac{44.16x 10^6 x 4x 0.006}{0.3} = 3.53 \text{ MN/m}^2$$