Glasgow Caledonian University

School of Engineering & Built Environment

MEng/BEng(Hons) in:

Mechanical-Electronic Systems Engineering
Mechanical & Power Plant Systems
Electrical Power Engineering
Computer-Aided Mechanical Engineering

Module: Engineering Design & Analysis 2
(Module No. M2H721926)

2D Stress Analysis: A Summary

Eur Ing **Professor M. Macdonald** BSc MSc PhD CEng FIMechE FIES FHEA
Department of Engineering
Room M203A
T: 0141 331 3540
E: mmd3@gcu.ac.uk

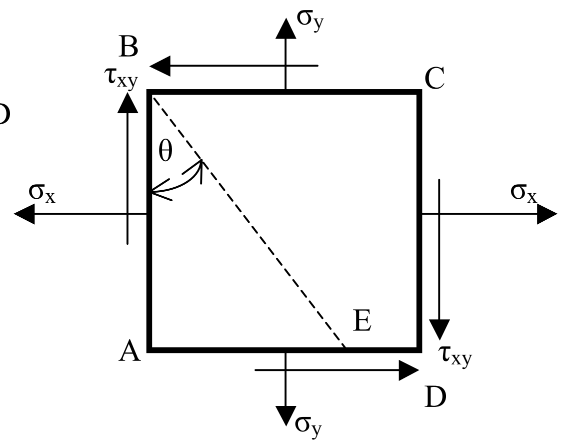
GLASGOW CALEDONIAN UNIVERSITY

School of Engineering & Built Environment

ENGINEERING DESIGN & ANALYSIS 2 (M2H721926)

2D Stress Analysis

The analysis of a general 2D stress system begins by considering the element shown subjected to direct stresses σ_x, σ_y and torsional stress τ_{xy}. Refer to the 2D Complex Stress System shown.



It is important to be able to calculate the stress σ_θ and τ_θ acting on a plane BE. Obviously the maximum σ or τ levels and the plane (θ) on which they act are most important. Consider therefore the element ABE, with thickness t.

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau \sin 2\theta \quad (\text{N/m}^2) \text{ [+ve tensile, -ve compression]}$$

$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau \cos 2\theta \quad (\text{N/m}^2)$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} \quad (\text{N/m}^2) \text{ [Major principal stress: +ve tensile, -ve compression]}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} \quad (\text{N/m}^2) \text{ [Minor principal stress: +ve tensile, -ve compression]}$$

These planes (σ_1 and σ_2) are inclined at an angle θ to the plane carrying σ_x as given by:

$$\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y} \quad [\theta \text{ in degrees}]$$

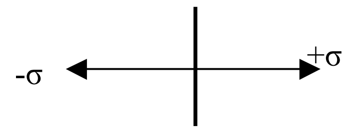
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$ (N/m^2) [***Note:*** τ_{max} occurs at 45° to σ_1 and σ_2]

Or, $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2}$ (N/m^2)

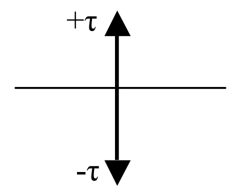
It can be seen that the equations for σ_1, σ_2, σ_θ, τ_{max} are equations that describe a circle having τ as its ordinate (y-axis) and σ as the abscissa (x-axis). This is known as the **Mohr's Circle of Stress** and allows for a graphical solution of 2D stress system problems.

Mohr's Circle of Stress – Method of Construction

1. All normal stresses (σ) are measured along the x-axis from the origin. Tensile stresses (+ve) to the right, compressive stresses (-ve) to the left.



2. Shear stresses (τ) are measured along the vertical axis: +ve shear measured above the origin, -ve shear below the origin.

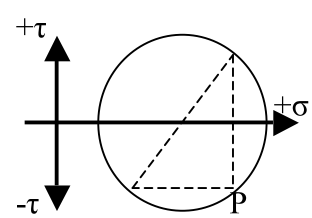


3. Any point on the circumference of the circle represents a plane in the material, and the coordinates of the point represent stresses in that plane.

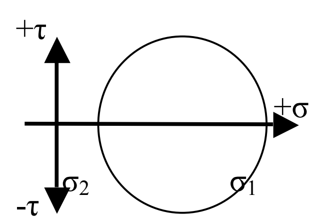
4. Two planes at right angles are represented by diametrically opposite points in the Mohr Circle.

5. The angle subtended at the centre of the circle by two points on the circumference is twice the angle between the two planes represented by these points.

6. The vertical line drawn through a point representing a plane meets the circle again at a point P called the Pole Point of the particular reference plane. The angle between this line and the line drawn from P to a point on the circumference of the circle representing any other plane, is the angle between the reference plane and the other plane. The relative positions shown in the stress plane are the true relative positions as they occur in the physical plane.



7. Two points at the intersection of the circle and the x-axis represent the principal planes in which the principal stresses act (σ_1, σ_2).



8. A line drawn from the origin to a point in the circumference representing a plane, gives the resultant stress on the plane. The angle between this line and the x-axis is the angle of obliquity of the resultant stress.

9. The centre of the Mohr Circle *always* lies on the x-axis.