Glasgow Caledonian University

School of Engineering & Built Environment

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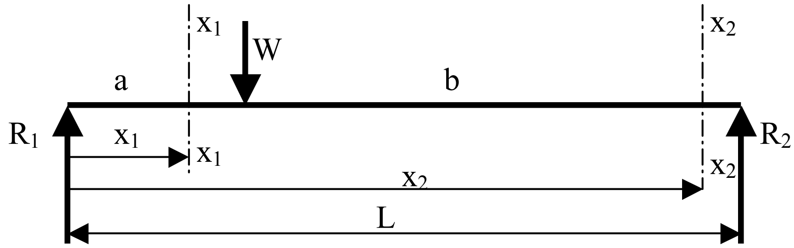
Deflection of Beams
Macaulay's Method: A Summary

Eur Ing **Professor M. Macdonald** BSc MSc PhD CEng FIMechE FIES FHEA
Department of Engineering
T: 0141 331 3540
E: mmd3@gcu.ac.uk

Macaulay's Method

Consider the non-standard simply supported beam with a concentrated off-centre load as shown Mccauley's Method is required to solve for deflection.



Consider section x_1x_1: $M_{x1x1} = R_1x_1$ and is valid for $0 < x < a$.

Consider section x_2x_2: $M_{x2x2} = R_1x_2 - W(x_2 - a)$ and is valid for $a < x < L$.

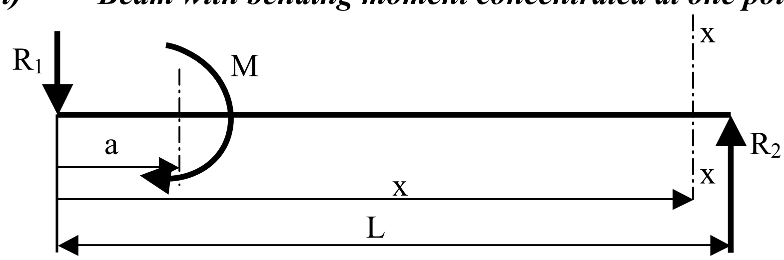
In order to use the elastic beam deflection equation $EI\frac{d^2y}{dx^2} = -M_{xx}$, the expression for M must hold for any section x-x from x = 0 to x = L. Necessary to ignore the term containing the bracket when the quantity within the bracket becomes negative, i.e. when $x < a$, $(x - a)$ is negative and should be ignored. Hence, $M_{xx} = R_1x - W(x - a)$ can be considered to be a general equation for the bending moment at any section along the beam.

Rules for Macaulay's Method:

1. Take the origin at either left or right hand extremity and section x-x just preceeding the last force action at the other end.
2. Express M_{xx} in terms of distances from the origin.
3. Substitute into $EI\frac{d^2y}{dx^2} = -M_{xx}$ and integrate keeping all bracket terms intact.
4. When evaluating the constants of integration, slope or deflection, any term for which the quantity within the brackets becomes negative is discarded.

Typical Applications of Macauley's Method.

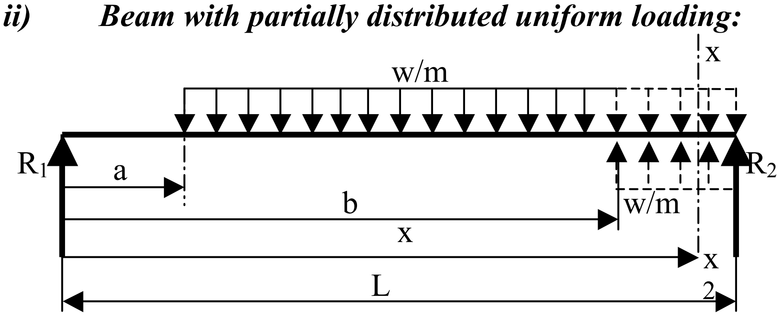
i) Beam with bending moment concentrated at one point:



If an applied moment M acts on the beam at any point other than the ends, then the distance 'a' from the origin must be included in the expression for M_{xx}. This is achieved by using the mathematical statement that any number raised to the power 0 (zero) is unity.

i.e. $M_{xx} = -R_1x + M(x-a)^0$, and if (x-a) term becomes negative, i.e. $(0<x<a)$, the term M(x-a) is ignored.

ii) Beam with partially distributed uniform loading:



$$\therefore \quad M_{xx} = R_1x - \frac{w}{2}(x-a)^2 + \frac{w}{2}(x-b)^2$$

It is easier to integrate this equation!