GLASGOW CALEDONIAN UNIVERSITY

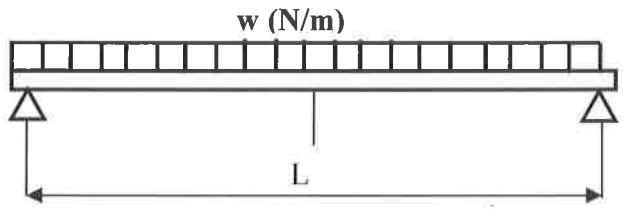
School of Engineering & Built Environment

ENGINEERING DESIGN & ANALYSIS 2 (M2H721926)

Tutorial: Deflection of Beams

1. For the 2 standard beam cases shown in Figure Q1, derive equations for deflection and slope at any point along each beam, and hence derive equations for the maximum values.

i)



ii)

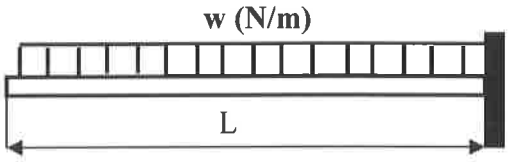


Figure Q1

2. A beam of length 10 m is simply-supported at A and B and it carries a load of 100 kN at midspan as shown in Figure Q2. The beam has a rectangular cross-section 75 mm x 250 mm. Taking E = 200 GN/m^2 for the beam material, calculate the maximum vertical deflection of the beam. **(106.7 mm)**

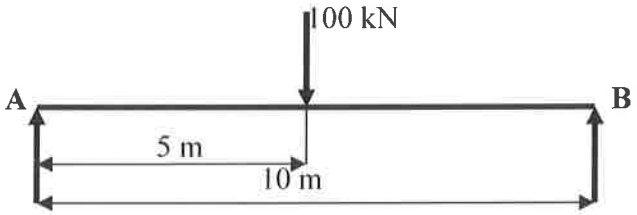


Figure Q2

3. A point load of 50 kN and a UDL of w = 10 kN/m are applied to a cantilever 8 m in length as shown in Figure Q3. The cantilever has a round cross-section of diameter 275 mm. Calculate the maximum vertical deflection at the free end. Take E = 205 GN/m^2. **(237.7 mm)**

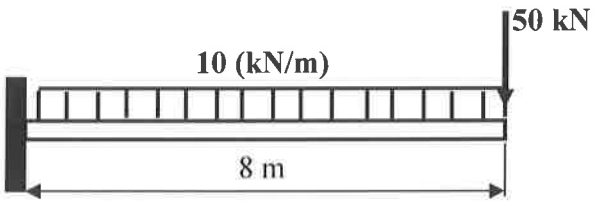


Figure Q3

4. The beam ABC is simply-supported at A and C and it carries a uniformly distributed load of 7.5 kN/m over its entire length and a concentrated load of 180 kN at the beam midspan as shown in Figure Q4. The beam cross-section is hollow rectangular of width 100 mm and depth 300 mm with a uniform wall thickness of 10 mm. Taking E = 208 GN/m^2 for the beam material, calculate the maximum vertical deflection of the beam. **(206 mm)**

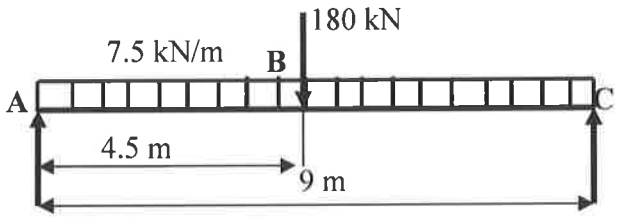


Figure Q4

5. The uniform cross-section cantilever of length L (m) shown in Figure Q5, carries a uniformly distributed load ω (N/m) and is supported on a rigid knife edge at its free end. Determine an equation for the supporting force P (N). Take the flexural stiffness of the cantilever = EI. **(P=3ωL/8)**

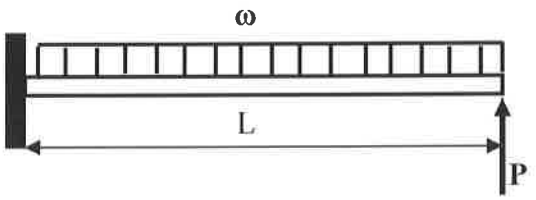


Figure Q5

6. A beam of length 10 m is simply-supported at A and B and it carries a load of 100 kN at a point 3 m from A as shown in Figure Q.6. Taking EI = 350 MNm^2 for the beam, calculate the vertical deflection at the point of loading. **(14.1 mm)**

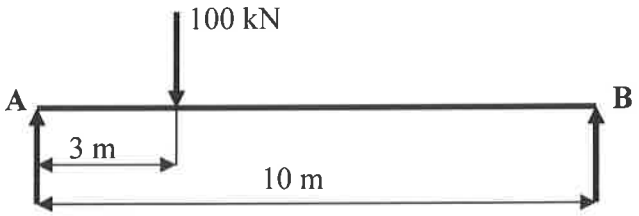


Figure Q6

7. The beam ABCD is simply-supported at A and D and it carries concentrated loads at B and C as shown in Figure Q7. Taking EI = 320 MNm^2 for the beam, calculate the slope at A and D, and the vertical deflection at B and C. **(0.0114 rad; -0.012 rad; 48.2 mm; 49.5 mm)**

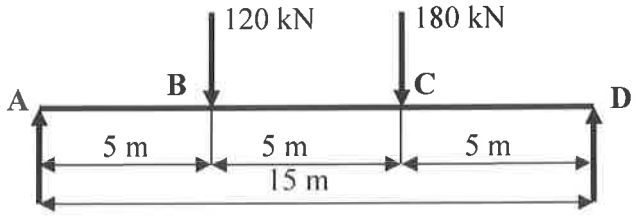


Figure Q7

8. The beam AB is simply-supported at A and B and is loaded as shown in Figure Q8. Taking EI = 215 GNm^2 for the beam, determine the slope and vertical deflection at each quarter point of the span of the beam. **(0.006 rad, 22.75 mm; -0.025 rad, 31.6 mm; -0.0058 rad, 20.03 mm)**

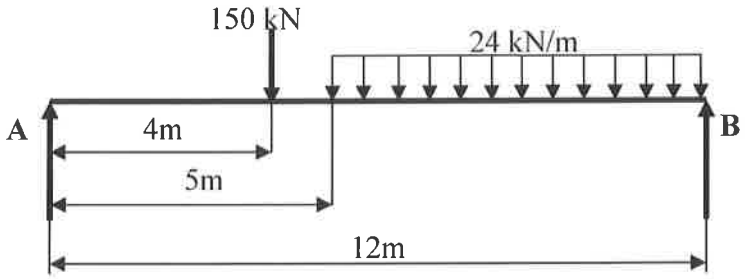


Figure Q8

9. The beam ABC is simply-supported at A and is continuous over the support B to C. When the beam carries the loading shown in Figure Q9, determine the slope at A, B and C, and the vertical deflection at the midpoint of AB and at C. Take EI = 60 GNm^2 for the beam, **(0.00106 rad; -0.00138 rad; 0.00129 rad; 18 mm; 2.18 mm)**

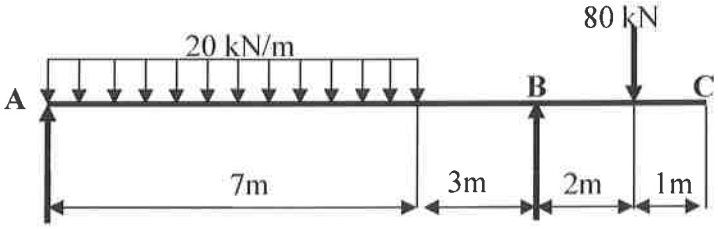


Figure Q9

10. A beam of uniform section shown in Figure Q10 has a length of 4 m and is simply-supported at points A and C. The beam carries a concentrated load P = 6 kN at B, a uniformly distributed load w = 4 kN/m on BC and an applied anticlockwise moment of 10 kNm at C. The flexural rigidity of the beam is EI = 3 x 10^3 kNm^2.
    - **(a)** Determine the reactions at A and C. **(R_A = 10 kN, R_C = 4 kN)**
    - **(b)** Draw the shear force and bending moment diagrams, highlighting the significant values.
    - **(c)** Determine the deflection of the beam at B. **(-3.26 mm)**
    - **(d)** Determine the deflection angle of the beam at C. **(-5.41x10^{-3} rad)**
    - **(e)** Determine the position and the values of the maximum deflection of the beam. **(-3.91 mm at x = 1.589 mm)**

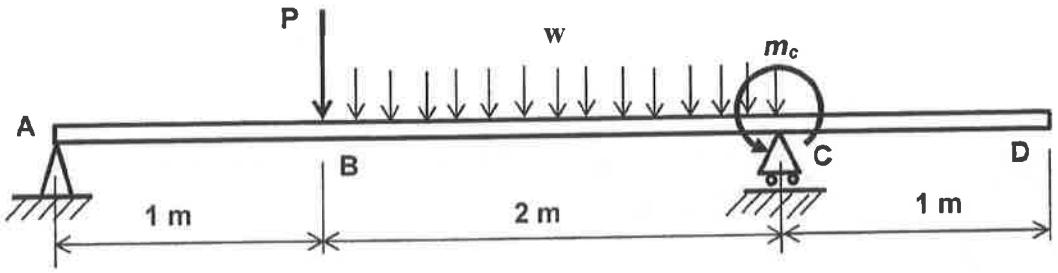


Figure Q10

11. A beam of uniform section shown in Figure Q11 has a length of 7 m and is simply-supported at points A and C. The beam carries a concentrated load P = 18 kN at B, a uniformly distributed load w = 5 kN/m on CD and an applied anticlockwise moment of 4 kNm at C. The flexural rigidity of the beam is EI = 3 x 10^4 kNm^2.

(a) Determine the reactions at A and C. **(R_A = 8 kN, R_C = 20 kN)**

(b) Draw the shear force and bending moment diagrams, highlighting the significant values.

(c) Determine the deflection of the beam at B and D. **(-2.25 mm; -38.2 mm)**

(d) Determine the deflection angle of the beam at A and C. **(1.15x10^{-3} rad; -9.5x10^{-4} rad)**

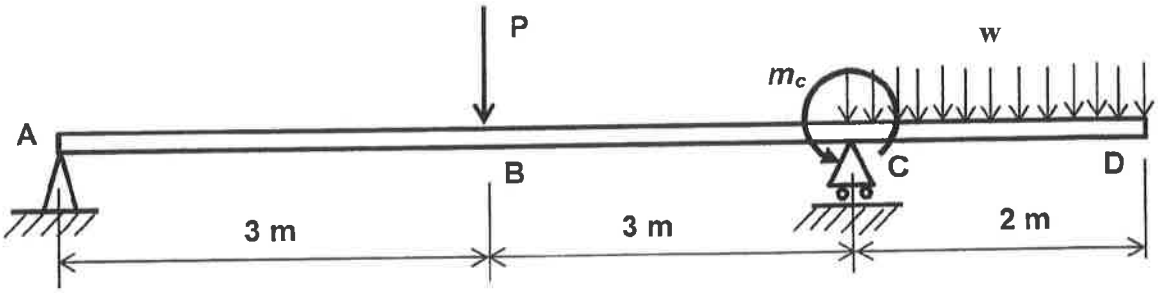


Figure Q11