**Worked Example No.1**
**Deflection of Beams**

For the simply-supported beam shown in Figure Ex.1, determine the maximum deflection and the maximum slope of the beam, and where they occur.

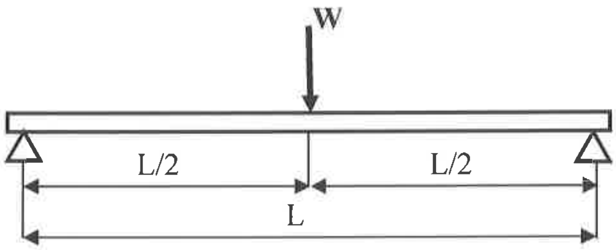
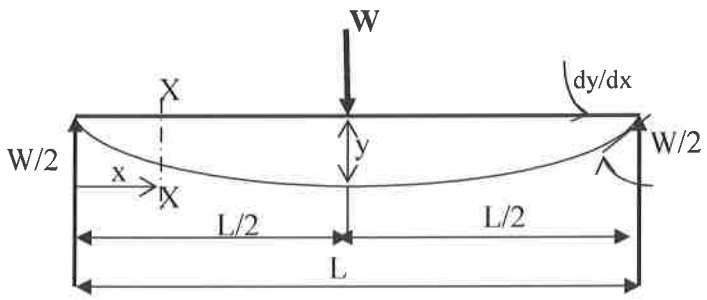



**Figure Ex.1**

***Solution***




**Direct Integration Method**
If the value of M at any point on a beam is known in terms of x, the distance along the beam, and provided that the equation applies along the complete beam, then integration of the differential equation of flexure Eq(3) will give deflections and slopes at any point, and, importantly, the maximum values and where they occur,

i.e. $$EI\frac{d^2y}{dx^2} = -M \quad \text{and} \quad \frac{dy}{dx} = \int \frac{M}{EI}dx + A \quad \text{(slope)}$$

or, $$y = \iint\left(\frac{M}{EI}dx\right)dx + Ax + B \quad \text{(deflection)}$$

where A and B are *constants of integration* evaluated from known conditions of slope and deflection for particular values of x (i.e. *boundary conditions*).

For the simply-supported beam with the concentrated point load at mid-span – taking the origin at the left-hand end, the bending moment at any section XX, distance x from the origin is:

$$M_{xx} = \frac{1}{2}Wx \quad \text{(where x < L/2)}$$

It is important to note that this expression and any integrals of it do not apply to the right-hand half of the beam. Substituting in the differential equation of flexure gives:

$$EI\frac{d^2y}{dx^2} = -M_{xx} = -\frac{1}{2}Wx$$

Integrating: $$EI\frac{dy}{dx} = -\frac{1}{4}Wx^2 + A \quad \text{(assuming EI is constant)} \qquad \text{(i)}$$

The constant of integration can be evaluated using the mid-span conditions: when x = L/2, slope dy/dx = 0. This gives:

$$0 = -\frac{1}{4}W\left(\frac{1}{2}L\right)^2 + A$$

$$\therefore \qquad A = \frac{WL^2}{16}$$

Substituting A in (i) and integrating again gives:

$$EIy = -\frac{Wx^3}{12} + \frac{WL^2x}{16} + B \qquad \text{(ii)}$$

Here, B = 0 since the deflection y = 0 at the origin (x = 0). The maximum deflection occurs at mid-span (x = L/2) and from (ii) is:

$$y_{\max} = \frac{1}{EI}\left[\frac{-W(L/2)^3}{12} + \frac{WL^2(L/2)}{16}\right] = \underline{\frac{WL^3}{48EI}}$$

From the above analysis, the slope at the left-hand support is found by substituting x = 0 in (i):

$$\left(\frac{dy}{dx}\right)_{\max} = \frac{A}{EI} = \underline{\frac{WL^2}{16EI}}$$