# School of Engineering & Built Environment

## MEng/BEng(Hons) in:

**Mechanical-Electronic Systems Engineering**
**Electrical Power Engineering**
**Mechanical & Power Plant Systems**
**Computer-Aided Mechanical Engineering**

## Module: Engineering Design & Analysis 2
***(Module No. M2H721926)***

# Euler Buckling Theory: A Summary

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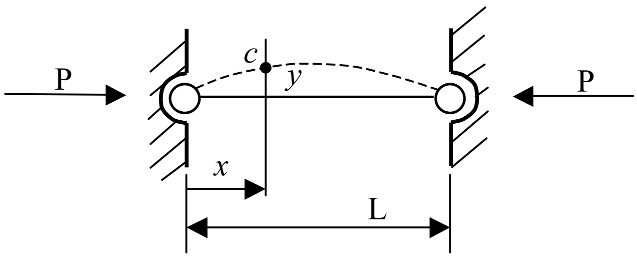
**ENGINEERING DESIGN & ANALYSIS 2 (M2H721926) – Failure of Components in Compression: Buckling**

## Euler Buckling Theory

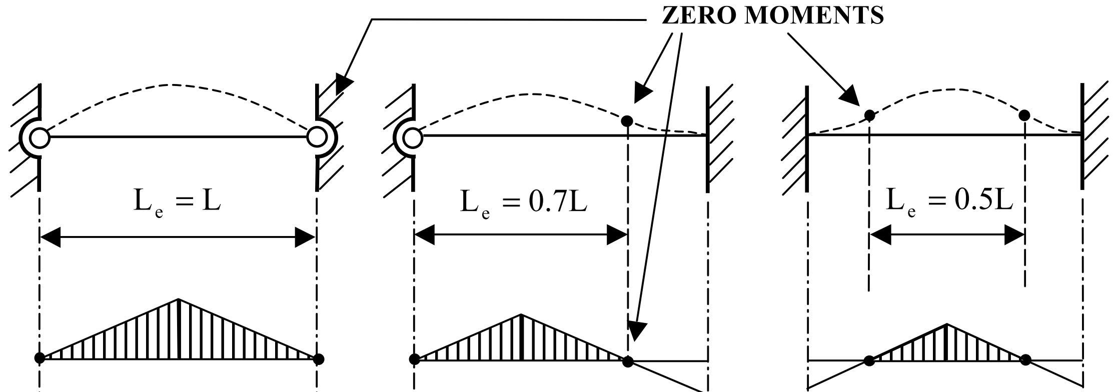
The ***mode*** of failure is different in compression than that of the tension failure and is termed *buckling*. Buckling occurs due to *out-of-plane-bending* instead of the material fracture experienced in a tensile test.


Normally engineering systems do not contain single compression members (struts) on their own, but are usually an assembly of differently loaded components. Because of the apparent weakness to compressive loading, these components require special attention in their analysis to avoid failure which occurs at load levels and stress levels much less than the strength of the materials being used.

### Pin-Ended Column Subject to Pure Axial Compression

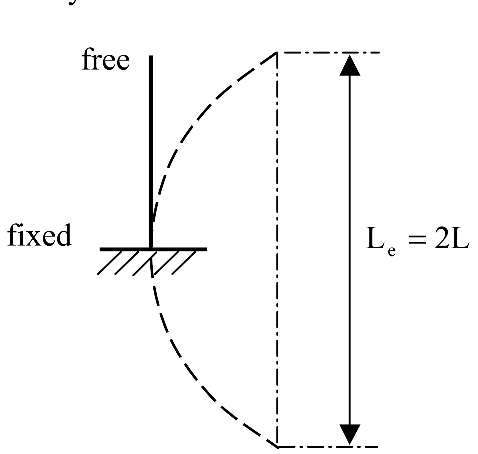


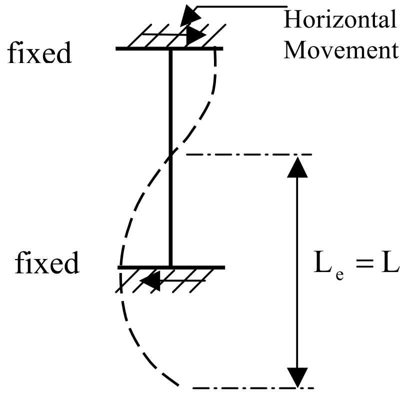

Buckling load capacity given by: $P = \dfrac{\pi^2 EI}{L^2}$ [Note: this equation applies to a pin-ended strut or column only]. What if the ends are not pinned? Use the concept of the ***equivalent length*** $\boldsymbol{L_e}$, e.g.




$$P_e = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EI}{L^2} \qquad = \frac{\pi^2 EI}{(0.7L)^2} = \frac{2\pi^2 EI}{L^2} \qquad = \frac{\pi^2 EI}{(0.5L)^2} = \frac{4.\pi^2 EI}{L^2}$$

Similarly:




* <u>3 ASSUMPTIONS MADE:</u>
- Strut initially straight
- Load purely axial
- Homogenous material

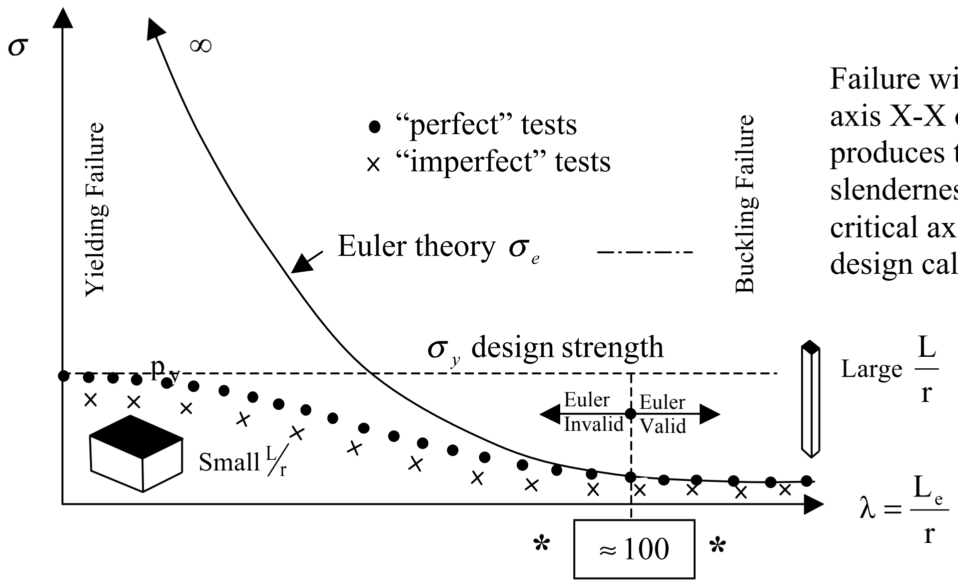
Hence, the ***Euler*** buckling load equation is: $P_e = \dfrac{\pi^2 EI}{L_e^2}$ (N)

**Euler Stress**

$$\sigma_e = \frac{P}{A} = \frac{\pi^2 EI}{L_e{}^2 A} = \frac{\pi^2 EAr^2}{L_e{}^2 A} = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2}$$

where $r = \sqrt{\dfrac{I}{A}}$ = ***radius of gyration***, and $\dfrac{L_e}{r}$ = ***slenderness ratio, λ***.

**Graph of Euler Stress v. Slenderness Ratio**




Failure will occur about the axis X-X or Y-Y which produces the **largest** (critical) slenderness ratio, $\lambda$. This critical axis is then used for design calculations.