Euler Buckling Theory
Worked Example No.1

A steel column of hollow rectangular cross-section has pinned ends and is subjected to an axial compressive load as shown in Figure Ex.1.

i) Determine the slenderness ratio for the column cross-section about both the X-X and Y-Y axes.

ii) Determine the magnitude of the critical axial load using Euler's theory, given that the Modulus of Elasticity for the steel is 200 GN/m^2

iii) Sketch an Axial Load vs. Slenderness Ratio curve and highlight all the significant values.

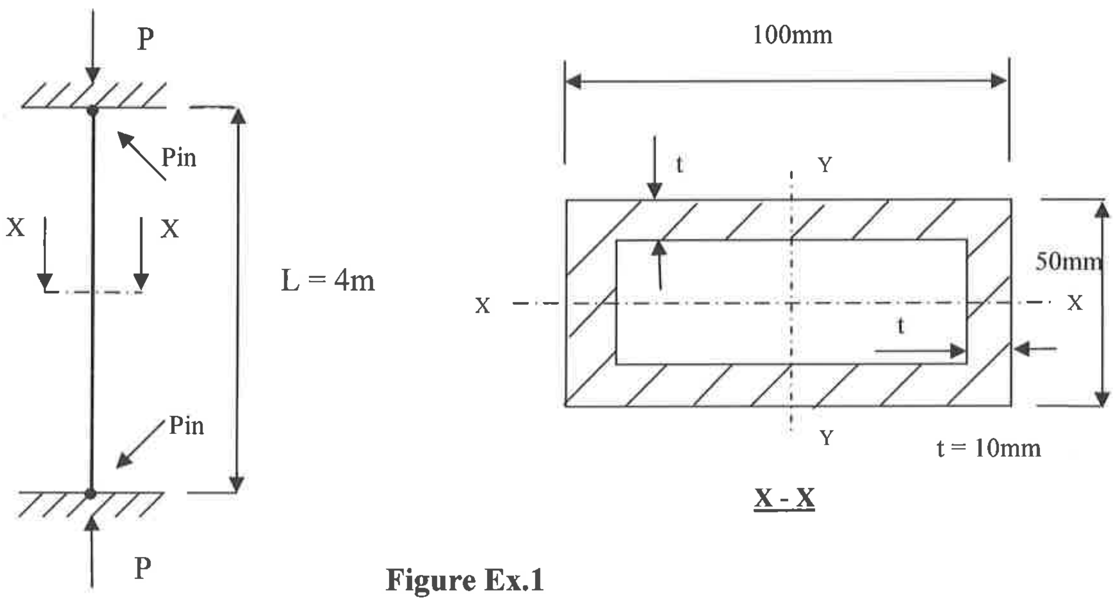
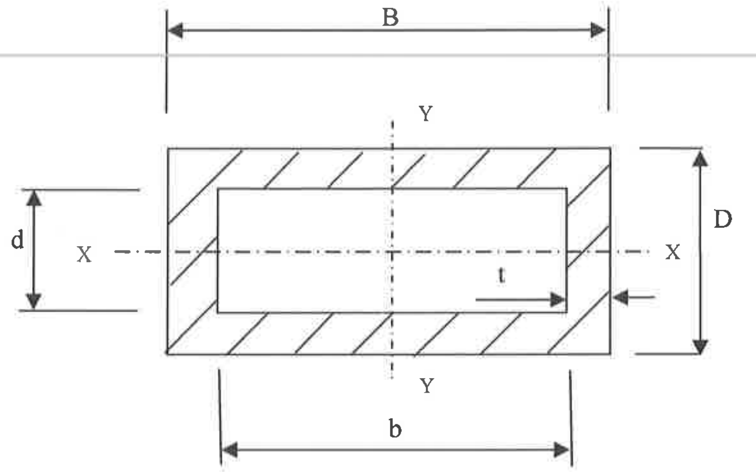


Figure Ex.1

Solution

i)



B = 100mm

D = 50mm

t = 10 mm

b = 80mm

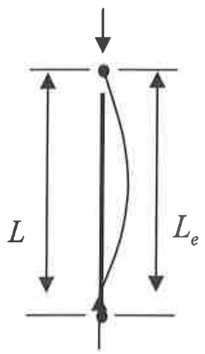
d = 30mm

$$Ixx = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{100\times 50^3}{12} - \frac{80\times 30^3}{12} = \underline{0.862\times 10^6\,mm^4}$$

$$Iyy = \frac{DB^3}{12} - \frac{db^3}{12} = \frac{50\times 100^3}{12} - \frac{30\times 80^3}{12} = \underline{2.887\times 10^6\,mm^4}$$

$$A = BD - bd = (100\times 50) - (80\times 30) = \underline{2600mm^2}$$

$$\therefore r_{xx} = \sqrt{\frac{Ixx}{A}} = \sqrt{\frac{0.862\times 10^6}{2600}} = \underline{18.21mm} \quad \text{and} \quad r_{yy} = \sqrt{\frac{Iyy}{A}} = \sqrt{\frac{2.887\times 10^6}{2600}} = \underline{33.32mm}$$



For a pin - ended strut, $L_e = L = 4m = \underline{4000mm}$

$$\therefore \lambda xx = \frac{L_e}{r_{xx}} = \frac{4000}{18.21} = \underline{219.7} \quad \text{and} \quad \lambda yy = \frac{L_e}{r_{yy}} = \frac{4000}{33.21} = \underline{120.4}$$

Hence the largest value of λ is the critical slenderness ratio :

$$\boxed{\lambda_{crit.} = \lambda_{XX} = 219.7}$$

This means that buckling will occur about the <u>weakest</u> axis, ie. X - X axis, where the I value is the <u>lowest</u> and hence provides least resistance to bending and hence buckling.

ii)

$E = 200 \times 10^9 \, N/m^2 = 200 \times 10^3 \, N/mm^2$

$Ixx = 0.862 \times 10^6 \, mm^4$

$Iyy = 2.887 \times 10^6 \, mm^4$

$L_e = 4000mm$

$P_{e_{crit.}} = ?$

$$P_e = \frac{\pi^2 EI}{L_e^2}$$

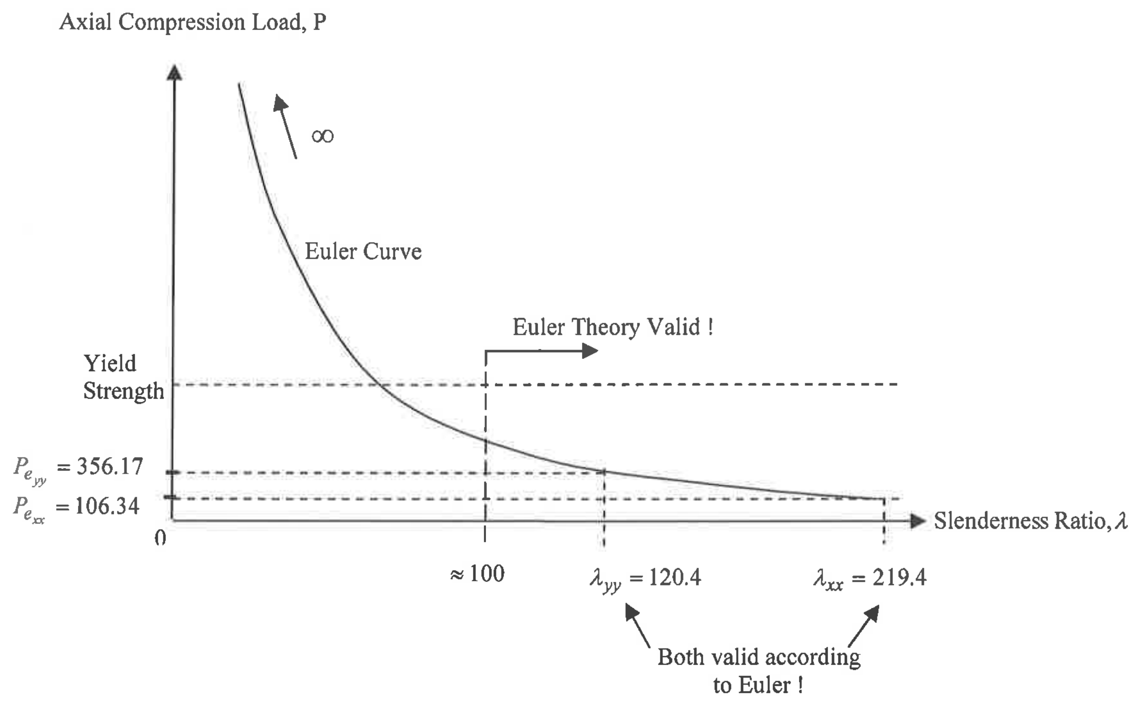
$$\therefore P_{e_{xx}} = \frac{\pi^2 \times 200 \times 10^3 \times 0.862 \times 10^6}{4000^2} = \underline{106.34kN}$$

$$P_{e_{yy}} = \frac{\pi^2 \times 200 \times 10^3 \times 2.887 \times 10^6}{4000^2} = \underline{356.17kN}$$

Hence, the critical axial load according to Euler Theory is $P_{e_{xx}}$.

i.e. $\boxed{P_{crit.} = P_{e_{xx}} = 106.34kN}$

iii)



Note: It is always useful to produce an axial compressive load v. slenderness ratio graph showing the Euler curve, the yield line and the validity limit of Euler.
In addition, the inclusion of the actual λ values obtained from a design problem along with the corresponding P values helps to illustrate Euler Theory and its proper application.