School of Engineering & Built Environment

MEng/BEng(Hons) in:

Mechanical-Electronic Systems Engineering
Mechanical & Power Plant Systems
Electrical Power Engineering
Computer-Aided Mechanical Engineering

Module: Engineering Design & Analysis 2
(Module No. M2H721926)

Dynamics: Damped Free Vibration – A Summary

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ENGINEERING DESIGN & ANALYSIS 2 (M2H721926)

Dynamics: Revision - Free Undamped Vibration

Systems that vibrate freely and undamped with one degree of freedom were analysed in Year 1. This is the simplest type of vibration problem - a single degree of freedom system having no damping where the effects of friction and all other resistances are ignored.

Figure 1 below shows the simplest type of single degree of freedom system represented by a mass m (kg) and elastic stiffness represented by a spring of stiffness k (N/m).

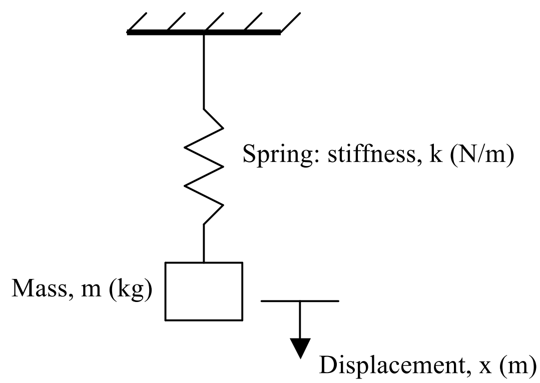


Figure 1

Equation of Motion for a one degree of freedom freely vibrating and undamped system, and its solution will give the displacement x in terms of time t is given by:

$$m\ddot{x} + kx = 0$$

where m = mass (kg)
k = stiffness (N/m)
x = displacement (m)
and $\ddot{x}$ = acceleration (m/s^2)

Natural frequency of vibration is given by: $\omega_n = \sqrt{\dfrac{k}{m}}$ (rad/s)

Natural frequency of vibration is also given by: $f_n = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} = \dfrac{\omega_n}{2\pi}$ (Hz)

The time taken for one cycle of vibration to occur is known as the *periodic time* and is given by: $T = \dfrac{1}{f_n}$ (s)

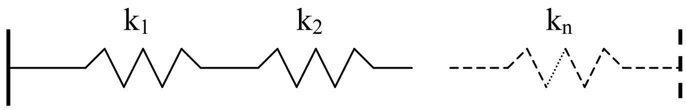
System Stiffness

To solve single degree of freedom vibration problems the stiffness of the system has to be known or calculated. For a system with one spring, the stiffness is defined as the ratio of force F (N) to deflection x (m), i.e.

$$\boxed{k = \frac{F}{x} \quad \text{(N/m)}}$$

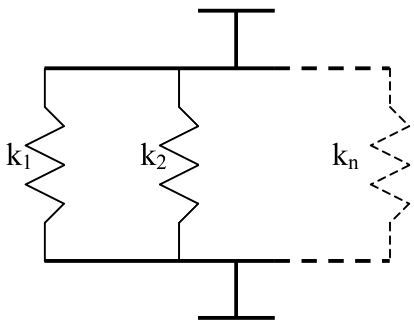
It is very common to find systems with more than one spring that can be arranged in series or in parallel and hence an *overall* or *total* system stiffness k_T has to be calculated.

Series spring connection:



$$\boxed{\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots \frac{1}{k_n}}$$

Parallel spring connection:



$$\boxed{k_T = k_1 + k_2 + \ldots k_n}$$

Hence, with the system total stiffness calculated, the natural frequency can be determined as follows:

$$\boxed{\omega_n = \sqrt{\frac{k_T}{m}} \quad \text{(rad/s)}}$$

Dynamics: Damped Free Vibration

During vibration, energy is dissipated in one form or another, and steady amplitude cannot be maintained without its continuous replacement.

Consider the simplest type of free vibration system with one single degree of freedom, having damping and where the effects of friction and all other resistances are ignored.

Figure 2 below shows a schematic diagram of this simplest of system represented by a mass m, elastic stiffness represented by a spring and viscous damping represented by a ‘dashpot’ as shown in Figure 3.

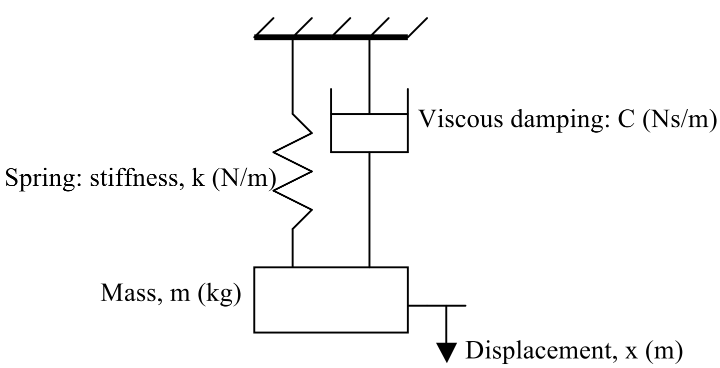
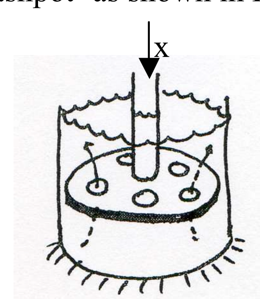


Figure 2



A typical dashpot configuration.

Figure 3

Equation of Motion for a one degree of freedom freely vibrating damped system, and the solution of the differential equation will give the displacement x in terms of time t and is given by:

$$m\ddot{x} + C\dot{x} + kx = 0 \qquad \text{or} \qquad \ddot{x} + \frac{C}{m}\dot{x} + \frac{k}{m}x = 0$$

where m = mass (kg); k = stiffness (N/m); C = viscous damping coefficient (Ns/m);
x = displacement (m); $\dot{x}$ = velocity (m/s) [= dx/dt]; and $\ddot{x}$ = acceleration (m/s^2) [= $d\dot{x}/dt$].

Critical damping coefficient: $C_c = 2m\omega_n$ (Ns/m)

Damping ratio: $\zeta = \dfrac{C}{C_c}$

Considering each of the 3 damping response cases, i.e.

1. $\zeta = 1$ (critically damped where $C = C_c$)
2. $\zeta > 1$ (overdamped where $C > C_c$)
3. $\zeta < 1$ (underdamped where $C < C_c$)

Theoretically the motion will not cease, however, in practice it does due friction and other effects. The frequency of the damped oscillation is given by:

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \quad \text{(rad/s)}$$

Logarithmic Decrement (or Decay)
As can be seen in the previously (in particular Case 3), the effect of viscous damping is to reduce the amplitude of vibration over time. The 'decay' or reduction in vibration amplitude can be calculated and is known as the '*logarithmic decrement*'.

$\ln\dfrac{x_1}{x_2}$ is called the logarithmic decrement and is denoted 'δ', i.e., $\delta = \ln\dfrac{x_1}{x_2}$ where x_1 is the initial amplitude of vibration.

$$\therefore \qquad \delta = \zeta\omega_n\tau \quad \text{and,} \quad \tau = \frac{1}{f} \text{ (s)}$$

$$\therefore \qquad \delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

Finally, for 'n' oscillations: $\delta = \dfrac{1}{n}\ln\dfrac{x_1}{x_n}$