## 4.10. Solved Problems – Belt Drive System Design

***Problem 1***

***Calculate the maximum power that may be transmitted by a flat belt driving a pulley 360 mm diameter which rotates at 180 rev/min. The maximum belt tension is 500 N and the angle of lap is 145°. The coefficient of friction between the belt and the pulley is 0.35.***

***Solution***

*Data:*
Pulley dia, d = 360 mm = 0.36m
∴ r = 0.18 m
N = 180 rev/min
Max. tension (tight side), $F_1$ = 500 N
Angle of lap, θ = 145°
μ = 0.35

Angle of lap, θ = 145° = (145/360) x 2π = 2.5307 rads

$$\frac{F_1}{F_2} = e^{\mu\theta} \quad \therefore \quad \frac{500}{F_2} = e^{0.35x2.5307} = 2.425 \quad \therefore \quad F_2 = 206.2 \text{ N}$$

$$\text{Power, } P = T\omega = (F_1 - F_2)r\omega = \frac{(F_1 - F_2).r.2\pi N}{60} = \frac{(500 - 206.2)x0.18x2\pi x180}{60} = \mathbf{997\ W}$$

***Problem 2***

***A multiple vee-belt drive is required to transmit 30 kW from a pulley 160 mm diameter rotating at 450 rev/min. The angle of lap is 170° and the groove angle is 40°. The coefficient of friction between the belt and the pulley is 0.35 and the maximum permissible stress in the belt material is 2.6 MNm$^{-2}$. If the cross-sectional area of each belt is 600 mm$^2$, calculate the minimum number of belts required for the drive system.***

***Solution***

*Data:*
Power, P = 30 kW = 30 x $10^3$ W
Pulley dia, d = 160 mm = 0.16 m
∴ r = 0.08 m
N = 450 rev/min
Angle of groove, α = 40°/2 = 20°
Angle of lap, θ = 170°
μ = 0.35
Belt material max stress, σ = 2.6 MNm$^{-2}$ = 2.6 Nmm$^{-2}$
Belt cross-sectional area, $A_b$ = 600 mm$^2$

Angle of lap, θ = 170° = (170/360) x 2π = 2.967 rads

$$\text{Power, } P = T\omega \qquad \therefore \quad T = \frac{P}{\omega} = \frac{60P}{2\pi N} = \frac{60x(30x10^3)}{2\pi x450} = 636.6 \text{ Nm}$$

T = $(F_1 - F_2)r$ $\quad\therefore\quad F_1 - F_2 = \frac{T}{r} = \frac{636.6}{0.08} = 7958$ N (1)

$$\frac{F_1}{F_2} = e^{\frac{\mu\theta}{\sin\alpha}} \quad \therefore \quad \frac{F_1}{F_2} = e^{\frac{0.35 x 2.967}{\sin 20^\circ}} = 20.83 \quad \therefore \quad F_2 = \frac{F_1}{20.83} \qquad (2)$$

Substitute (2) in (1): $F_1 - \frac{F_1}{20.83} = 7958$ $\quad\therefore\quad$ $F_1 - 0.048F_1 = 7958$ $\therefore$ $0.952F_1 = 7958$

$\therefore$ $F_1 = 8359$ N (i.e. total tight side tension)

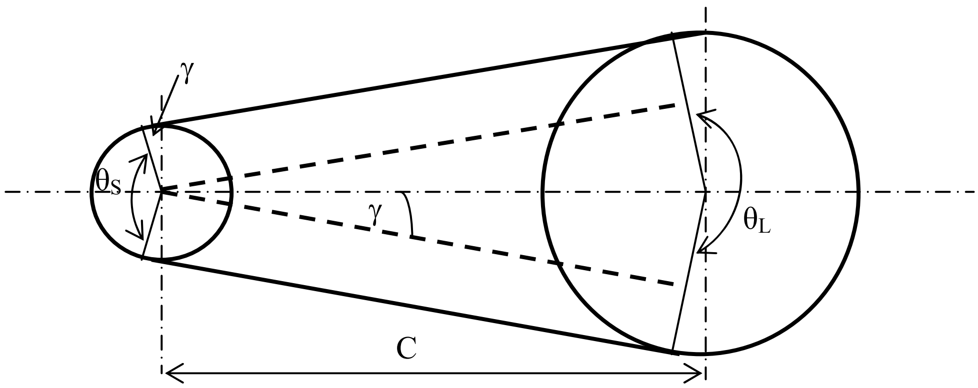
Now, $F_1 = \sigma A = 2.6 \text{ x } 600 = 1560$ N

Minimum number of belts required = 8359/1560 = 5.36

Hence **6 belts** will be required!

***Problem 3***

***A multiple vee-belt drive is required to transmit 24 kW from a pulley 150 mm in diameter to another pulley 300 mm in diameter rotating at 300 rev/min. The centre distance (C – see the figure below) is 1000 mm and the groove angle is 40°. The coefficient of friction between the belt and the pulley is 0.31 and the maximum permissible belt tension is 500 N. For this geometry and conditions, calculate the minimum number of belts required for the drive system.***




***Solution***

*Data:*

Power, P = 24 kW = $24 \text{ x } 10^3$ W

Small Pulley dia, $d_1$ = 150 mm = 0.15 m

$\therefore$ $r_1 = 0.075$ m

Large Pulley dia, $d_2$ = 300 mm = 0.30 m

$\therefore$ $r_2 = 0.15$ m

$N_2 = 300$ rev/min

Angle of groove, $\alpha = 40°/2 = 20°$

$\mu = 0.31$
Belt material max permissible tension, $F_{max} = 500$ N

Belt angle, $\gamma$ (from the geometry)

$$\sin\gamma = (r_2 - r_1)/C$$
$$= (150 - 75)/1000$$
$$\Rightarrow \gamma = \sin^{-1}(0.075)$$
$$= 4.30^0$$
$$= 0.075\,rad$$

Angle of lap, $\theta_S = \pi - 2.\gamma = 2.9915$ rads

Speed of small pulley, $N_1 = N_2 \times \frac{d_2}{d_1} = 300 \times \frac{0.30}{0.15} = 600\,rpm$

Power, P = Tω $\quad \therefore \quad T = \frac{P}{\omega} = \frac{60P}{2\pi N_1} = \frac{60x(24x10^3)}{2\pi x600}$ = 381.97 Nm

T = ($F_1 - F_2$)r $\quad \therefore \quad F_1 - F_2 = \frac{T}{r_1} = \frac{381.97}{0.075}$ = 5092.96 N (1)

$$\frac{F_1}{F_2} = e^{\frac{\mu\theta}{\sin\alpha}} \quad \therefore \quad \frac{F_1}{F_2} = e^{\frac{0.31x2.9915}{\sin 20^\circ}} = 15.05 \quad \therefore \quad F_2 = \frac{F_1}{15.05} \qquad (2)$$

Substitute (2) in (1): $F_1 - \frac{F_1}{15.05} = 5092.96 \quad \therefore \quad F_1 - 0.0664F_1 = 5092.96$

$\therefore \quad 0.9336F_1 = 5092.96$

$\therefore \quad F_1 = 5455.42$ N (i.e. total tight side tension)

Minimum number of belts required = 5455.42/500 = 10.91

Hence **11 belts** will be required!