



School of Engineering & Built Environment

African Leadership College Notes

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Module: Engineering Design and Analysis 2

Clutch Design and Belt Drives Summary

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11A. Clutch Design

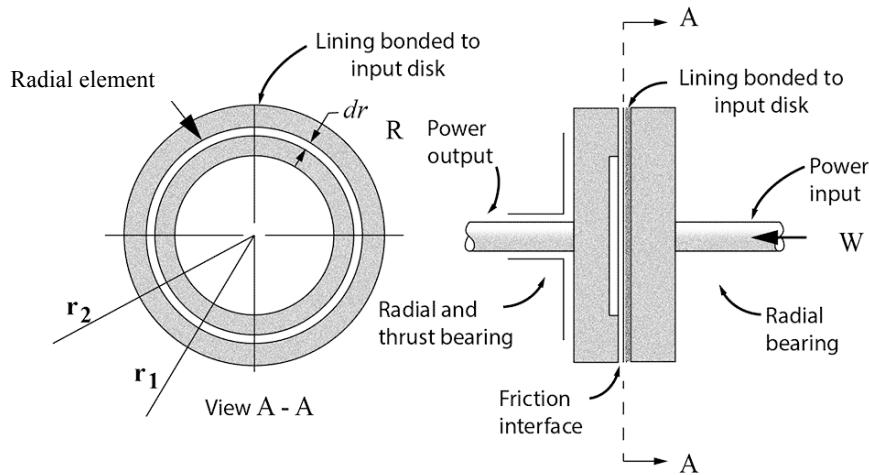


Figure 11A.1: A typical plate clutch arrangement.

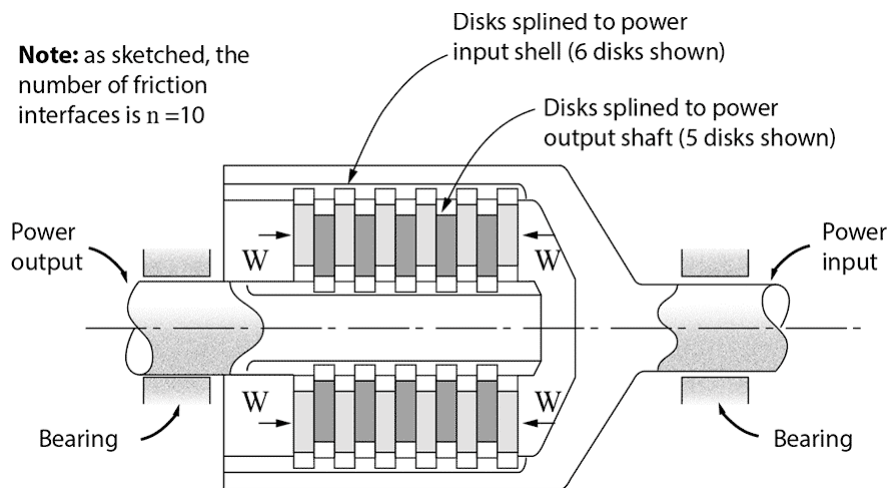


Figure 11A.2: A typical multi-plate clutch arrangement.

For the case of **Uniform Pressure**: it is assumed that the pressure is uniform over the contact area, hence p is a constant giving:

$$W = \pi \cdot p (r_1^2 - r_2^2) \therefore p = \frac{W}{\pi (r_1^2 - r_2^2)}$$

and,

$$T = \frac{2}{3} \pi \cdot \mu \cdot p (r_1^3 - r_2^3)$$

Subs for p gives:

$$T = \frac{2}{3} \cdot \mu \cdot W \frac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)} \quad (\text{Nm}) \quad (11A.1)$$

For the case of **Uniform Wear**: when clutch plates are new the pressure at the contact faces will tend to be uniform.

$$T = \mu WR \text{ (Nm)} \quad (11A.2)$$

Note: For multi-plate clutches, equations (11A.1) and (11A.2) are multiplied by 'n'.

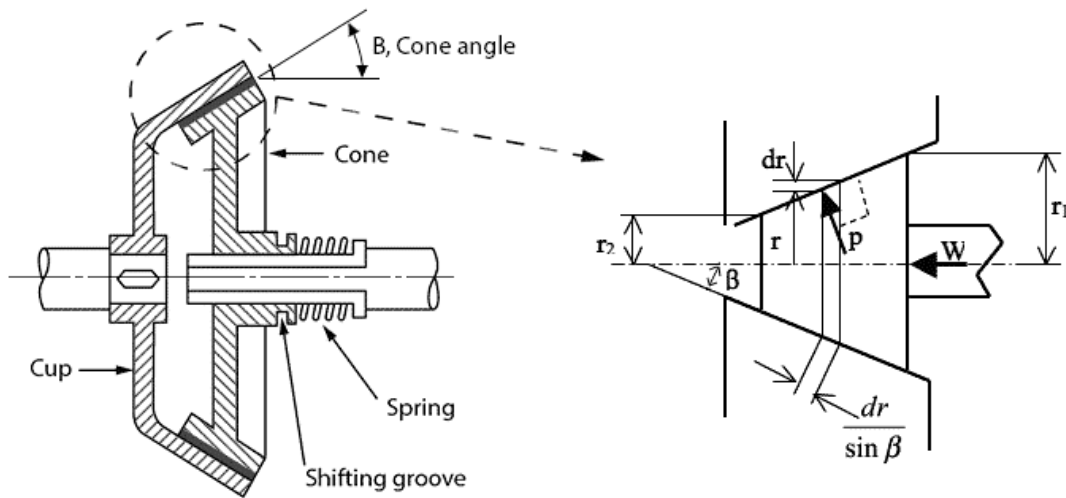


Figure 11A.3: A typical cone clutch arrangement.

$$T = \frac{2}{3} \cdot \frac{\mu \cdot W}{\sin \beta} \frac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)} \text{ (Nm)} \quad (11A.3)$$

Assuming p is constant, i.e. **uniform pressure**:

Finally, assuming $p \cdot r$ is constant, i.e. **uniform wear**:

$$T = \frac{\mu \cdot W}{2 \sin \beta} (r_1 + r_2)$$

$$\therefore T = \frac{\mu \cdot WR}{\sin \beta} \text{ (Nm)} \quad (11A.4)$$

Nomenclature for Clutches

Symbol	Description	Unit
A	Contact area	m ²
dr	Small portion of contact disc	m
n	Number of contact pairs (plate clutch)	-
p	Intensity of pressure between contact surfaces	Nm ⁻²
R	Mean radius	m
r ₁	Outer radius	m
r ₂	Inner radius	m
T	Transmitted torque	Nm
W	Axial thrust force	N
β	Cone angle	degrees

μ	Coefficient of friction	-
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11B. Belt Drive Analysis

Power Transmission by a Belt Drive

Figure 11B.1 shows a portion of a continuous belt in contact with a drive pulley of effective radius r . The tension F_1 is greater than tension F_2 due to friction between the belt and the pulley.

F_1 is referred to as the tension in the 'tight side' of the belt; F_2 is referred to as the tension in the 'slack side' of the belt.

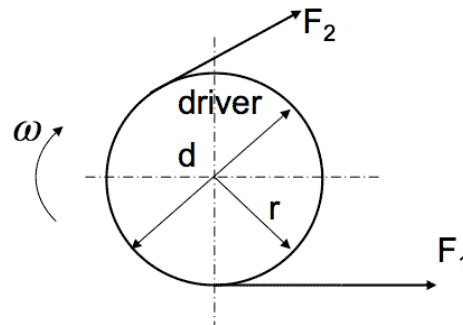


Figure 11B.1: Forces on a belt around a pulley

The pulley drives the belt in a clockwise direction and, provided there is no slip, the peripheral speed of the pulley will be equal to the linear speed of the belt.

$$\text{Net clockwise torque on pulley } T = F_1.r - F_2.r = (F_1 - F_2)r$$

$$\text{Now, Power } P = T\omega \quad \therefore \quad P = (F_1 - F_2)r\omega$$

where ω = angular velocity of pulley (rads^{-1})
 r = effective radius of pulley (m)
 F_1 = tension in tight side of belt (N)
 F_2 = tension in slack side of belt (N)

The difference in tensions ($F_1 - F_2$) is referred to as the *effective tangential force*.

$$\text{Also, note that since } v = \omega r, \text{ then: } P = (F_1 - F_2)v \quad (\text{W}) \quad (11B.1)$$

where v = linear speed of belt (m/s).

Ratio of Belt Tensions

If the belt velocity is constant, the power transmitted will depend upon the difference in the belt tensions ($F_1 - F_2$). Hence if the ratio (F_1/F_2) can be made greater, then more power will be transmitted. However when the ratio (F_1/F_2) reaches a certain value, known as the *limiting ratio*, the belt will slip on the pulley. It thus seems likely that (F_1/F_2) depends upon:

- the coefficient of friction (μ) between the belt and the pulley;
- the angle of lap (θ) of the belt round the pulley, and that an increase in either μ and θ will lead to an increase in the ratio (F_1/F_2)

It can be shown that the limiting ratio of tensions (F_1/F_2) is given by:

$$\frac{F_1}{F_2} = e^{\mu\theta} \quad (11B.2)$$

where, F_1 = tension in tight side of belt (N)
 F_2 = tension in slack side of belt (N)
 e = constant = 2.718 (base of natural logarithms)
 μ = coefficient of friction between belt and pulley
 θ = angle of lap of belt round pulley (rads)

Note that where an open belt connects two pulleys of different diameters, the angle of lap on the smaller pulley will be less than that on the larger pulley. From Figure 4.5 it can be seen that θ_s is less than θ_L .

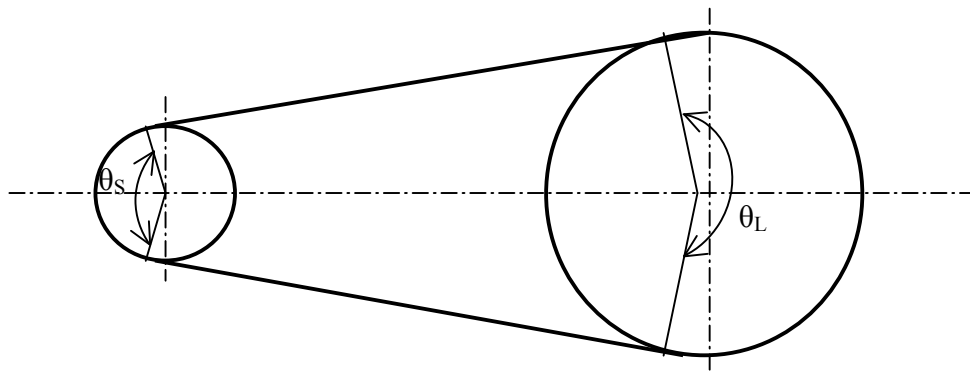


Figure 11B.2: Angle of lap for drive and driven pulley

Calculations, then, would be based on the smallest pulley.

Vee-Belt Drives

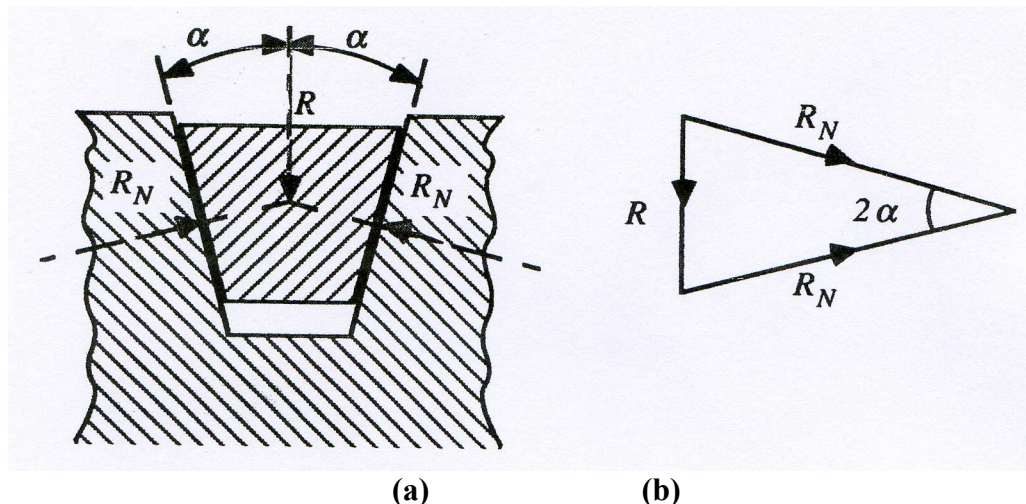


Figure 11B.3: Vee belt and pulley interaction

The belt tension sets up a radial force R acting towards the centre of the pulley. From Figure 11B.3(b):

$$\sin \alpha = \frac{R/2}{R_N} \quad \therefore \quad \frac{R}{2} = R_N \sin \alpha$$

$$\therefore R = 2R_N \sin \alpha \quad \therefore \quad 2R_N = \frac{R}{\sin \alpha}$$

Now, for flat belts, the friction force = μR

and for vee-belts, the friction force = $2\mu R_N$ (two friction surfaces) = $\frac{\mu R}{\sin \alpha}$

Hence, the belt tension formula for flat belts, $\frac{F_1}{F_2} = e^{\mu\theta}$ is modified for vee-belts and becomes:

$$\frac{F_1}{F_2} = e^{\frac{\mu\theta}{\sin \alpha}}$$

where, F_1 = tension in tight side of belt (N)

F_2 = tension in slack side of belt (N)

e = constant = 2.718

μ = coefficient of friction between belt and pulley

θ = angle of lap of belt round pulley (rads)

and, 2α = included angle of pulley groove ($^\circ$)

Nomenclature for Belt Drives

Symbol	Description	Unit
d	Pulley diameter	m
$d\theta$	Angle of small section of belt	degrees
e	Base of natural log = 2.718	-
F	Force on stationary pulley	N
F_0	Initial belt tension	N
F_1	Tight side tension	N
F_2	Slack side tension	N
$F_1 - F_2$	Effective tangential force	N
F_1/F_2	Ratio of belt tensions (limiting)	-
F_C	Centrifugal tension	N
m	Mass of belt section	kg
N	Rotational speed	rpm
P	Power	W
r	Pulley (effective) radius	m
R	Radial force	N
R_N	Force normal to groove surface	N
T	Drive torque	Nm
v	Linear belt speed	ms^{-1}
α	Groove half angle	degrees
γ	Belt angle	rad
μ	Coefficient of friction between belt and pulley	-
θ	Angle of lap	degrees
ω	Rotational velocity	rads^{-1}
	Subscripts	
L	Large Pulley	
S	Small Pulley	