Chapter 16

Laplace Transforms: the s domain

16.1 Method

Tables of Laplace transforms are used to manipulate (add, multiply, subtract and divide) complex differential equations, and the obtain a solution to the equations.

The procedure is as follows:

1. Transform each term in the differential equation into its Laplace transform, i.e. change the function of time (t) to the function of (s).
2. Carry out all algebraic manipulations, e.g 'step input' times 'system transfer function' equals a resulting 'output ' function.
3. Convert the resulting output Laplace function back into an equation giving a function of time, i.e. invert the Laplace transform equation.

16.2 Questions

1. Determine, using the table of Laplace transforms, the Laplace transform of the following time domain equations (i.e. convert from time domain

to Laplace domain.)

(a) A step voltage of size 4 V which starts at $t = 0$.
(b) A step voltage of size 4 V which starts at $t = 2$ s.
(c) A ramp voltage which starts at $t = 0$ and increases at the rate of 3 V/s.
(d) A ramp voltage which starts at $t = 2$ s and increases at the rate of 3 V/s.
(e) An impulse voltage of size 4 V which starts at $t = 3$ s.
(f) A sinusoidal voltage of amplitude 2 V and angular frequency 10 Hz.

2. Determine, using the table of Laplace transforms, the inverse Laplace transforms of the following expressions (i.e. convert from Laplace domain to time domain.

(a)
$$t^2$$

(b)
$$t^2 e^{-at}$$

(c)
$$t^2(1 + e^{-at})$$

3. Determine, using the table of Laplace transforms, the inverse Laplace transforms of the following expressions (i.e. convert from Laplace domain to time domain).

(a)
$$\frac{2}{s}$$

(b)
$$\frac{3}{2s+1}$$

(c)
$$\frac{2}{s-5}$$

4. Use Laplace transforms to solve the following differential equation.

$$3\frac{dx}{dt} + 2x = 4$$

with initial condition $x = 0$ at $t = 0$.

5. For a voltage step input of size v at $t = 0$ into a series RC circuit the differential equation for the potential difference across the capacitor v_c is given by

$$v = RC\frac{dv_c}{dt} + v_c$$

Use Laplace transforms to solve this equation and find an expression for $v_C(t)$.

6. For a step input at $t = 0$ of size v into a series LR circuit the current variation with time is described by the equation

$$\frac{L}{R}\frac{di}{dt} + i = \frac{v}{R}$$

The current $i = 0$ at $t = 0$. Using Laplace transforms, solve this equation.

7. When a mercury-in-glass thermometer is inserted into a hot liquid there is essentially a step input of temperature θ_i to the thermometer, where θ_i is the temperate of the hot liquid. The relationship between the output of the thermometer θ_o, i.e its reading, and time is given by the first order differential equation

$$K\frac{d\theta_o}{dt} = \theta_i - \theta_o$$

where θ_o is a function of time. Use the Laplace transformation to obtain a solution for this equation. Take the value of θ_o to be 0 at $t = 0$, i.e. in this problem we are only concerned with the change in temperature when the thermometer is inserted into the liquid. Thus θ_i is the temperature before it is inserted into the liquid.

8. Determine, using the table of Laplace transforms, the Laplace transform of the following time domain equations (i.e convert from time domain to Laplace domain)

   (a) A step voltage of size 6 V which starts at $t = 0$ s.

(b) A step voltage of size 6 V which starts at $t = 3$ s.
(c) A ramp voltage of 6 V/s which starts at $t = 0$ s.
(d) A ramp voltage of 6 V/s which starts at $t = 3$ s.
(e) An impulse voltage of size 6 V which starts at $t = 0$ s.
(f) An impulse voltage of size 6 V which starts at $t = 3$ s.
(g) A sinusoidal voltage of amplitude 6 V and angular frequency 50 Hz.

9. Determine, using the table of Laplace transforms, the Laplace transform of the following time domain equations (i.e convert from time domain to Laplace domain).

   (a)
   $$e^{-2t}$$
   (b)
   $$5e^{-2t}$$
   (c)
   $$v_o e^{-\frac{t}{\tau}}$$
   (d)
   $$1 - e^{-2t}$$
   (e)
   $$5\left(1 - e^{-2t}\right)$$
   (f)
   $$v_o\left(1 - e^{-\frac{t}{\tau}}\right)$$

10. Determine using the table of Laplace transforms. The inverse Laplace transforms of the following equations (i.e. convert from Laplace domain to time domain).

   (a)
   $$\frac{2}{(s+3)}$$
   (b)
   $$\frac{2}{(3s+1)}$$

(c)

$$\frac{2}{s(s+3)}$$

(d)

$$\frac{2}{s(3s+1)}$$

11. Solve the following differential equations.

(a)

$$2\frac{dx}{dt} + 5x = 6$$

where $x = 0$ at $t = 0$

(b)

$$8\frac{dx}{dt} + x = 4$$

where $x = 0$ at $t = 0$

12. Determine the Laplace transforms of the following voltages which vary with time according to the given equations.

(a)

$$v = 5(1 - e^{-\frac{t}{50}})$$

(b)

$$v = 10 + 5(1 - e^{-\frac{t}{50}})$$

(c)

$$v = 5e^{-\frac{t}{50}}$$