




| Description     | Time domain   | Laplace domain   |
|-----------------|---|--|
| Definition      | $f(t) = \mathcal{L}^{-1}\{F(s)\}$                   | $F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st} dt$  |
| Linearity       | $g(t) = af(t)$                                      | $G(s) = aF(s)$   |
|                 | $g(t) = f(t) + h(t)$                                | $G(s) = F(s) + H(s)$   |
| Differentiation | $g(t) = \frac{df(t)}{dt}$                           | $G(s) = sF(s) - f(0)$  |
|                 | $g(t) = \frac{d^2f}{dt^2}(0)$                       | $G(s) = s^2F(s) - sf(0) - \frac{df(t)}{dt}$  |
|                 | $g(t) = \frac{d^n f(t)}{dt^n}$                      | $G(s) = s^n F(s) - s^{n-1}f(0) - s^{n-2}\frac{df}{dt}(0) - \dots - s^2\frac{d^{n-3}f}{dt^{n-2}}(0) - s\frac{d^{n-2}f}{dt^{n-2}}(0) - \frac{d^{n-1}f}{dt^{n-1}}(0)$ |
| Integration     | $g(t) = \int_0^{\infty} f(t) dt$                    | $G(s) = \frac{1}{s}F(s)$   |
|                 | $g(t) = \int_0^{\infty} \int_0^{\infty} f(t) dt dt$ | $G(s) = \frac{1}{s^2}F(s)$   |
| Unit step       | $f(t) = 1$  |  $F(s) = \frac{1}{s}$   |
| Unit ramp       | $f(t) = t$  |  $F(s) = \frac{1}{s^2}$   |
| Unit parabola   | $f(t) = t^2$  |  $F(s) = \frac{2}{s^3}$   |

Continued ...

| Description | Time domain                       | Laplace domain  |
|-------------|-----------------------------------|---|
| Definition  | $f(t) = \mathcal{L}^{-1}\{F(s)\}$ | $F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st} dt$ |

Powers of  $t$

$$f(t) = t^n$$



$$F(s) = \frac{n!}{s^{(n+1)}}$$

Exponential decay

$$f(t) = e^{-at}$$



$$F(s) = \frac{1}{(s+a)}$$

$$f(t) = te^{-at}$$



$$F(s) = \frac{1}{(s+a)^2}$$

$$f(t) = t^2e^{-at}$$



$$F(s) = \frac{2}{(s+a)^3}$$

$$f(t) = t^n e^{-at}$$



$$F(s) = \frac{n!}{(s+a)^{(n+1)}}$$

Pure delay  $T$

$$f(t) = g(t - T)$$

$$F(s) = e^{-sT}G(s)$$

Delayed unit step

$$f(t) = \begin{cases} 1 & \text{if } t > T > 0 \\ 0 & \text{otherwise} \end{cases}$$



$$F(s) = \frac{e^{-sT}}{s}$$

Unit impulse

$$f(t) = \begin{cases} 1 & \text{if } t = 0 \\ 0 & \text{otherwise} \end{cases}$$



$$F(s) = 1$$

Continued ...

| Description | Time domain                       | Laplace domain  |
|-------------|-----------------------------------|---|
| Definition  | $f(t) = \mathcal{L}^{-1}\{F(s)\}$ | $F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st} dt$ |

Rectangular pulse  $f(t) = \begin{cases} 1 & \text{if } 0 < t < T \\ 0 & \text{otherwise} \end{cases}$



$$F(s) = \frac{1 - e^{-sT}}{s}$$

Sinusoid  $f(t) = \sin(\omega t)$



$$F(s) = \frac{\omega}{s^2 + \omega^2}$$

$$f(t) = \cos(\omega t)$$



$$F(s) = \frac{s}{s^2 + \omega^2}$$

$$f(t) = e^{-at} \sin(\omega t)$$



$$F(s) = \frac{\omega}{(s + a)^2 + \omega^2}$$

$$f(t) = e^{-at} \cos(\omega t)$$



$$F(s) = \frac{s + a}{(s + a)^2 + \omega^2}$$

$$f(t) = 1 - \cos(\omega t)$$



$$F(s) = \frac{\omega^2}{s(s^2 + \omega^2)}$$

Compound signals  $f(t) = 1 - e^{-at}$



$$F(s) = \frac{1}{s} - \frac{1}{s + a} = \frac{a}{s(s + a)}$$

$$f(t) = \frac{1}{a} \left( t - \frac{1 - e^{-at}}{a} \right)$$



$$F(s) = \frac{1}{s^2(s + a)}$$

$$f(t) = \frac{1}{a^2} (1 - e^{-at} - ate^{-at})$$






$$F(s) = \frac{1}{s(s + a)^2}$$

$$f(t) = (1 - at) e^{-at}$$



$$F(s) = \frac{s}{(s + a)^2}$$

Continued ...

| Description  | Time domain   | Laplace domain  |  |
|--------------|---|---|--|
| Definition   | $f(t) = \mathcal{L}^{-1}\{F(s)\}$   | $F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st} dt$                       |  |
|              | $f(t) = \frac{(e^{-at} - e^{-bt})}{b - a}$  |  | $F(s) = \frac{1}{(s + a)(s + b)}$        |
|              | $f(t) = 1 - \frac{b}{b - a}e^{-at} + \frac{a}{b - a}e^{-bt}$  |  | $F(s) = \frac{ab}{s(s + a)(s + b)}$      |
|              | $f(t) = \frac{e^{-at}}{(b - a)(c - a)} \cdots$ $\cdots - \frac{e^{-bt}}{(c - a)(a - b)} \cdots$ $\cdots + \frac{e^{-ct}}{(a - c)(b - c)}$ |  | $F(s) = \frac{1}{(s + a)(s + b)(s + c)}$ |
| Second order | $f(t) = \frac{\omega}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega t} \sin(\omega\sqrt{1 - \zeta^2}t)$   | $F(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$                         |  |
|              | $f(t) = 1 + A_1e^{P_1t} + A_2e^{P_2t}$  | $F(s) = \frac{\omega^2}{s(s^2 + 2\zeta\omega s + \omega^2)}$                      |  |

where  $a, T, \omega \in \mathbb{R}$  and  $n \in \mathbb{N}$  are constants;  $t, \tau \in \mathbb{R} \geq 0$  are time; and  $s \in \mathbb{C}$  is the Laplace operator; and

$$A_1 = -\frac{1}{2} - \frac{\zeta}{2\sqrt{\zeta^2 - 1}}$$

$$A_2 = -\frac{1}{2} + \frac{\zeta}{2\sqrt{\zeta^2 - 1}}$$

$$P_1 = -\zeta\omega - \omega\sqrt{\zeta^2 - 1}$$

$$P_2 = -\zeta\omega + \omega\sqrt{\zeta^2 - 1}$$