Solutions of Tutorial 6

Q1

The two programs are Feed in tariff and renewable energy certificates.

(i)

Feed-in Tariffs (FiT) are a financial incentive to support distributed and small-scale renewable energy generation (up to 5 MW in the UK)

FiT is available for the following generation technologies: Wind power - Solar PV power - Hydro power - Anaerobic digestion - Micro-CHP (≤ 2kW)

There are three sources of financial benefit from a generation project receiving FiTs:

Generation tariff:

A fixed price for each unit of electricity generated by the renewable system.

Export tariff:

A guaranteed price for each unit of electricity exported to the grid.

Import reduction:

Reducing your import from the grid by using your own electricity.

(ii)

Renewable Energy Certificate (REC) is a concept brought forward by regulators to incentivise the installation of more renewable energy plants by requesting licensed electricity suppliers to buy a certain percentage of their supply from eligible renewable generation plants.

When a renewable energy generator — a wind farm or solar power plant, for example — generates a megawatt-hour (MWh) of power, it receives one REC, a certificate saying that it generated one MWh of electricity from clean sources, which it can also sell in the open market.

This means every renewable generator has two revenue streams, electricity sold at prevailing market rates, and REC sold at rates set by the regulator (normally updated annually).

Suppliers prove compliance to the regulator by presenting a sufficient number of RECs to cover their obligation.

Suppliers can purchase RECs directly from renewable power plants, or from other suppliers. So, RECs is in reality a commodity.

Q2

(i) When no bidirectional meter installed, it is assumed 50% of generated power is exported to grid

Generation revenue = 4700*21.65/100 = £1017.55

Export revenue = 4700/2*4.64/100 = £109.04

Consumed solar energy = 4700/2 = 2350kWh

Savings = 2350*11/100 = £258.5

Net annual benefit = £1017.55+ £109.04 + £258.5 = £1385.09

(ii)

Generation revenue = 4700*21.65/100 = £1017.55

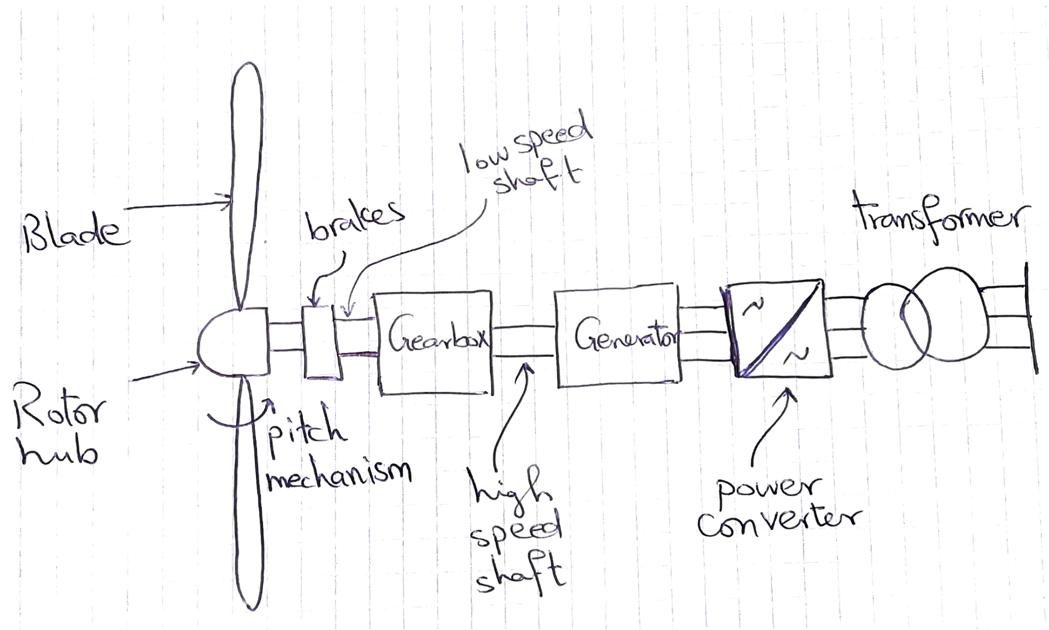
Export revenue = 1700*4.64/100 = £78.88

Consumed solar energy = 4700-1700 = 3000kWh

Savings = 3000 *11/100 = £330

Net annual benefit = £1017.55+ £78.88 + £330 = £1426.43

Q3



Blades: Capture wind power. They are designed to generate high lift-to-drag force ratio to turn the rotor hub efficiently.

Pitch mechanism: Rotates the blades around their axes to limit power capture in high winds. They are also used as aerodynamic brakes when it is required to stop the turbine.

Brakes: mechanical brakes activated at standstill to ensure that any lift force generated on the blades does not rotate the rotor hub and drive train.

Low speed shaft: couples the rotor hub to the gearbox.

Gearbox: Used to introduce speed transmission between the low speed rotor hub and high speed electric generator.

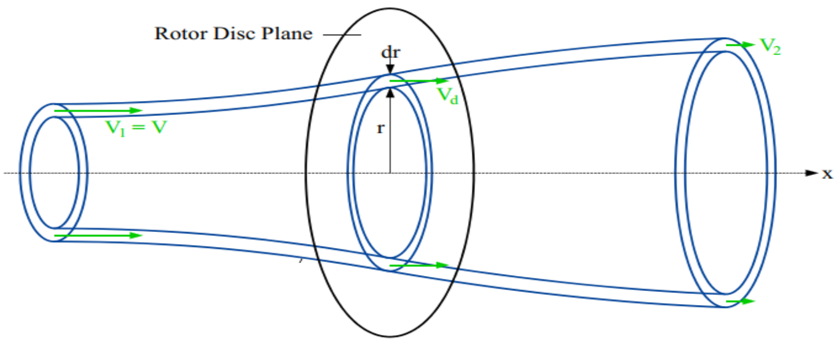
High speed shaft: couples the gearbox and brakes to the electrical generator

Generator: converts the mechanical rotation of the high speed shaft to electricity; can be an induction machine or synchronous machine.

Power converter: used in variable speed wind turbines to control the operation of the turbine and to convert the variable-frequency power generated by the generator to fixed-frequency power for transmission to supply.

Transformer: steps up the voltage from generator voltage level (typically 690V or 575V to collection network voltage, typically 33kV.

Q4



The mean wind speed at the WT rotor plane is:

$$v_d = \frac{v_1 + v_2}{2}$$

Wind mass flow rate at the WT rotor plane:

$$\dot{m} = \rho A vd = \rho A \frac{v_1 + v_2}{2}$$

Wind power at the WT rotor plane:

$$Pd = P_{w1} - P_{w2} = \frac{1}{2}\dot{m}(v1^2 - v2^2)$$

$$= \frac{1}{2}\rho A \frac{v_1 + v_2}{2} * v1^2\left(1 - \left(\frac{v2}{v1}\right)^2\right) = \frac{1}{2}\rho A v1 \; * \frac{1}{2}\left(1 + \frac{v2}{v1}\right) v1^2\left(1 - \left(\frac{v2}{v1}\right)^2\right)$$

$$Pd = \frac{1}{2}\rho A v1^3 * \frac{1}{2}\left(1 + \frac{v2}{v1}\right)\left(1 - \left(\frac{v2}{v1}\right)^2\right) = Pw1cp$$

The performance coefficient (c_p)

$$c_p = \frac{1}{2}\left(1 + \frac{v_2}{v1}\right)\left(1 - \left(\frac{v2}{v1}\right)^2\right) \qquad (1)$$

The speed ratio for maximum value of c_p can be found by identifying the local maxima of c_p function of equation (1) with respect to speed ratio ($\frac{v_2}{v1}$) through the first derivative of c_p function

$$\frac{dc_p}{d(v_2/v_1)} = 0$$

$$\therefore \frac{d}{d\left(\frac{v_2}{v_1}\right)}\left[\frac{1}{2}\left(1+\frac{v_2}{v_1}\right)\left(1-\left(\frac{v_2}{v_1}\right)^2\right)\right]=0$$

rename $\frac{v_2}{v_1}=x$ for simplicity

$$\therefore \frac{d}{dx}\left[\frac{1}{2}(1+x)(1-x^2)\right]=0$$

$$\therefore \frac{d}{dx}\left[\frac{1}{2}-\frac{1}{2}x^2+\frac{1}{2}x-\frac{1}{2}x^3\right]=0$$

$$\therefore -x+\frac{1}{2}-\frac{3}{2}x^2=0$$

multiply both sides by -2 and rearrange

$$\underbrace{3}_{a}x^2+\underbrace{2}_{b}x-\underbrace{1}_{c}=0$$

$$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-2\pm\sqrt{4+12}}{6}=\frac{-1\pm 2}{3}$$

$$\therefore x=\frac{1}{3} \quad \text{or} \quad x=-1$$

$$\therefore \boxed{\frac{v_2}{v_1}=\frac{1}{3}} \quad \text{or} \quad \frac{v_2}{v_1}=-1 \Rightarrow$$ has no physical meaning

$$\therefore @ \frac{v_2}{v_1}=\frac{1}{3} \Rightarrow C_{P_{max}}=\frac{1}{2}\left(1+\frac{1}{3}\right)\left(1-\frac{1}{9}\right)=\frac{1}{2}*\frac{4}{3}*\frac{8}{9}$$

$$\boxed{C_{P_{max}}=\frac{16}{27}} \Rightarrow$$ Betz limit

Q5

(i) Maximum aerodynamic efficiency occurs when $\frac{v_2}{v1} = \frac{1}{3}$
so outgoing speed v_2 = 12/3 = 4m/s.

(ii)

output mechanical power from WT @ max aero. eff.

$$P_m = P_w * C_{p_{max}} = \frac{1}{2} \rho A v_w^3 * C_{p_{max}}$$
$$= \frac{1}{2} \rho \pi R^2 v_w^3 * C_{p_{max}}$$

$$\therefore P_m = \frac{1}{2} * 1.225 * \pi * \left(\frac{82}{2}\right)^2 * (12)^3 * 0.593$$

$$\boxed{P_m \cong 3.313 \text{ MW}}$$

(iii)

(iii) Operation at $\lambda_{opt} \Rightarrow C_{p_{max}} = 0.47$

$$P_e = P_w * C_{p_{max}} * \eta_{gear} * \eta_{gen}$$
$$= \frac{1}{2} \rho A v_w^3 * C_{p_{max}} * \eta_{gear} * \eta_{gen}$$
$$= \frac{1}{2} * 1.225 * \pi (41)^2 * 12^3 * 0.47 * 0.8 * 0.92$$

$$\boxed{P_e \cong 1.933 \text{ MW}}$$

overall WT efficiency

$$\eta_{WT} = C_{p_{max}} * \eta_{gear} * \eta_{gen} = 0.47 * 0.8 * 0.92 * 100\%$$

$$\boxed{\eta_{WT} = 34.6\ \%}$$

(iv) While the wind turbine could theoretically capture over 3MW of power from the incoming wind at 12m/s, the aerodynamic design limitations as well as mechanical and electrical losses in the drivetrain and generator pushed this figure down to below 2MW as the overall wind turbine efficiency becomes short of 35%.

Q6

N_r = 15 rpm   A = 7850 m^2

(i) $\lambda = \frac{\omega_r R}{v_w} \Rightarrow v_w = \frac{\omega_r R}{\lambda}$

to find v_w, ω_r and R and λ must be found

$$\omega_r = \frac{2\pi N_r}{60} = \frac{2\pi * 15}{60} = 1.57 \text{ rad/s}$$

$$A = \pi R^2 \Rightarrow R = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{7850}{\pi}} = 50 \text{ m}$$

Since the WT operates at the MPPT curve, it means operation at optimal tip-speed ratio.
from Table Q6 $\Rightarrow$ λ_{opt} corresponds to C_{P_max}

$\therefore \lambda_{opt} = 8.5$

$$\therefore v_w = \frac{\omega_r R}{\lambda_{opt}} = \frac{1.57 * 50}{8.5} = 9.24 \text{ m/s}$$

(ii) $P_w = \frac{1}{2}\rho A v_w^3 = \frac{1}{2} * 1.225 * 7850 * 9.24^3$

$P_w = 3.787$ MW

Operation @ MPPT:

$$P_m = P_w C_{P_{max}} = 3.787 * 10^6 * 0.43 = 1.629 \text{ MW}$$

(iii)

WT overall efficiency when operating @ MPPT

Note $C_{p_{max}} = \eta_{aero_{max}}$

$$\eta_{WT} = \eta_{aero_{max}} * \eta_{mech} * \eta_{elec}$$

$$= 0.43 * 0.82 * 0.93 * 100\%$$

$$\boxed{\eta_{WT} = 0.327 = 32.7\ \%}$$

$$P_e = P_w * \eta_{WT} = P_m * \eta_{mech} * \eta_{elec}$$

$$\therefore P_e = 3.787 * 10^6 * 0.327$$

$$\boxed{P_e = 1.238\ \text{MW}}$$