Background to differentiation

Objectives:

- Appreciate that differentiation is a method to find instantaneous gradients
- See how differentiation can be formally defined

Key points:

This topic, with formal definitions of derivatives will not be assessed, but a background understanding and appreciation of its importance is very helpful later.

Differentiation is a method to find the rate of change of a function, it tells us how quickly a function is currently changing. When you differentiate a function you get its **derivative**. This derivative is another function which tells you the gradient of the starting function all the way along.

For graphed functions of x the derivative at $x = 3$ is a numerical value representing the **gradient** of the graph when $x = 3$. A gradient of 0 means the graph is (at least temporarily) horizontal, a negative gradient means it's going down, and positive gradient means it's going up. Larger sizes of derivative mean the gradient is steeper.

In practice, the derivative often represents a key quantity of a system.

- (Mechanics example) If we start with a function for position of a moving object then the derivative becomes the velocity (rate of change of position).
- (Electrostatics examples) Using Coulomb's Law, if we start with electrostatic potential due to a point charge, we can use the derivative to find the field strength.
- (Electromagnetism example) Using Faraday's Law, we can calculate an induced emf from the derivative of magnetic flux.

In engineering you normally have access to a table showing the derivatives of popular functions to avoid too much tedious memorization, but it never hurts to remember some.

There is lots of equivalent notation for the derivative of a function. If the function is called $f(x)$ then we can write $\frac{\mathrm{d}f}{\mathrm{d}x}$ (called *dee-eff-by-dee-ex*), f' (called f-prime or f-dash). Or $\frac{\mathrm{d}y}{\mathrm{d}x}$ and y' if you have called the function y by writing $y = f(x)$.

Recommended links:

Highly recommended: Khan Academy long course (can skip Estimating Derivative section, skip Differentiability, stop in the sine and cosine section after skipping the proofs), Mathcentre handout (Basic introduction handout via gradients, for engineers)

Other links: HELM notes (shows first principles derivations, quite mathematical)