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The Chain Rule for differentiating

Objectives:

⋄ Identify when you need to use the chain rule to differentiate

⋄ Learn how to use the chain rule for finding derivatives

Key points:

This topic is an introduction to the Chain Rule. This is basically just a method for finding the derivative of a function when **you can spot** it can be written as a **function of a function**, sometimes also called a **composite function**. As long as you can differentiate the two functions!

A lot of functions you come across are of this format, some simple examples would be:

$$f(x) = \sin(3x + 1) \quad \text{or} \quad v(x) = (2x + 3)^3 \quad \text{or} \quad w(x) = e^{1+x^2}$$

Being able to identify a function as one function applied to another is key before you try and use the chain rule. The chain rule can be written in two different popular ways. In the example of f above, you can spot that $f(x) = g(h(x))$ if **you choose** g to be the sine function, and **you choose** h to be the function which multiplies by 3 and adds 1. i.e. if $g(t) = \sin(t)$ and $h(t) = 3t + 1$ then

$$g(h(x)) = \sin(h(x)) = \sin(3x + 1) \quad \text{Remember: } g \text{ just takes the sine of whatever it is given}$$

Once we have that $f(x) = g(h(x))$ then these are the two ways to write the chain rule:

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{\mathrm{d}g}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}x} \text{ or perhaps more helpfully } f'(x) = g'(h(x)) \times h'(x)$$

I prefer the second formula, you need to find the derivative of g and put $h(x)$ inside, then multiply by the derivative of h with the main variable (x in this case) inside.

Here $g'(t) = \cos(t)$, so we get $f'(x) = \cos(h(x)) \times (3x + 1)' = \cos(3x + 1) \times 3 = 3\cos(3x + 1)$.

Beware: there are different rules (product rule and quotient rule) if the function is a **product of functions** or **fraction/quotient of functions**, like $x^2 \sin(x)$ or $\frac{\log(x)}{x}$ respectively. We don't cover these yet, some textbooks/notes cover them at the same time as the chain rule.

Recommended links:

Highly recommended: Mathcentre handout (Simple clear, though a little short)

Highly recommended: Khan Academy module (Nice video, interactive multipart lesson. Much more material here, perhaps too much at this stage)

Other links: Mathcentre notes