Second derivatives and rates of change

Objectives:

- Know how to use first and second derivatives to determine position and nature of turning/stationary points
- Be able to interpret derivatives in real problems as 'rates of change' as a variable changes

Key points:

There are two related topics here, they are both about using derivatives to answer questions.

The derivative of a function we are given is called its **first derivative**. If we differentiate this first derivative then we get the **second derivative** of our starting function and so on. The first and second derivatives of function f with variable x can be written as $\frac{df}{dx}$ and $\frac{d^2f}{dx^2}$, respectively. Or as f' and f''. Note that we write $\frac{d^2f}{dx^2}$ NOT the very messy $\frac{d\frac{df}{dx}}{dx}$ although they mean the same thing.

For graphs of functions, we often care about high points and low points (called maxima and minima, the plurals of maximum and minimum). To find where these are we **solve for x to find when $\mathbf{f'(x) = 0}$**. These points are called **stationary points** or **turning points**.

However, both $\cup$-shaped and $\cap$-shaped curves will have $f'(x) = 0$ in the middle. We must look at the **second derivative to work out if a stationary point is a minimum ($\cup$) or a maximum ($\cap$)**.

My advice is to visualise a $\cup$-shaped graph... the gradient **starts negative** then **reaches zero** then **becomes positive**. This is an example of a graph whose **gradient/derivative is increasing**. An increasing derivative, means the 'derivative of the derivative' (the second derivative) is **positive**, and you've found a minimum. At a maximum, the second derivative will be **negative**.

'Rate of change' problems are mostly about situations with many variables. You choose the variable you want to explore and differentiate the formula for the quantity you care about *with respect to that variable*. So it's **not always x**. If you care about how velocity, v, changes over time then you calculate $\frac{dv}{dt}$. If you care about power, P, as you vary some area A, then you need $\frac{dP}{dA}$ to understand how P depends upon A.

Recommended links:

Highly recommended: HELM notes (Key definitions), HELM notes (Introduction to maxima and minima, with engineering examples)

Highly recommended: Khan Academy: rates of change (see HELM for 2nd derivative definition)

Other links: Mathtutor notes on maxima and minima (basic definitions, but then moves on to optimisation immediately)