Optimisation via differentiation

Objectives:

- Practice examples of tackling real-life problems using differentiation

Key points:

This topic is linked with the topic on 'rates of change'. It can be quite a difficult topic as it requires practice in trying to convert a wordy description of a problem into algebra and then back to the real-world.

Always try and identify which variables **you can control/change or just naturally varies (like time)** and which variables change **as a consequence of other changes**.

A variable **you can control/change** is one to differentiate with respect to, i.e. on the bottom of the derivative symbol (typically x in other topics).

For example, if you have a formula for temperature, T,

$$T = 3e^{-at}\sin(\omega t), \text{ where } a \text{ and } \omega \text{ are constants, and } t \text{ is time.}$$

If you can draw a sketch of the graph, (T against t here) then do!

Then by studying $\frac{\mathrm{d}T}{\mathrm{d}t}$ you can find when T is maximal and minimal, and the answer will depend upon a and ω.

If you find a formula for T^* (when the temperature is a maximum) **and you can actually choose/control the constant a** then you could calculate

$$\frac{\mathrm{d}T^*}{\mathrm{d}a}$$

to try and understand how the maximum temperature depends upon a, and perhaps even find the 'best' choice of a when this maximum T^* is largest (or smallest) over all a choices.

Recommended links:

Highly recommended: HELM notes (Excellent introduction, with engineering examples)

Recommended: Mathtutor notes (Mathematical introduction to optimisation)

Other links: Khan Academy worked example (Making a box), Khan Academy worked example (Folding a box)