

Integration: indefinite and definite integrals

Objectives:

- Appreciate that integration can be seen as both anti-differentiation, and also a method for finding areas under curves
- Learn what definite and indefinite integrals are, and the difference between them

Key points:

Integrating is another procedure, like differentiating, that you can do to a function. The function you try and integrate, let's call it f, is called the **integrand** and the result from integrating f is called the **integral of f**.

A **definite integral** is written like this:

$$\int_1^3 x^2 + 1 \, \mathrm{d}x$$

An **indefinite integral** is written like this:

$$\int x^2 + 1 \, \mathrm{d}x$$

There are three parts...

- ♦ the \int_a^b part which tells you to integrate and if there are limits what they are. The definite integral says to **integrate from** *a* **to** *b* (here it says from 1 to 3)
- ♦ the integrand/function to integrate, $x^2 + 1$ in both the examples here
- ◊ the dx part which tells you the variable to use for the integration

As anti-differentiation, since $g(x) = \frac{1}{3}x^3 + x + C$ for any constant *C* has derivative $g'(x) = x^2 + 1$ then we say *g* is the indefinite integral of $f(x) = x^2 + 1$. It's any function whose derivative is *f*.

To find an area under a curve, we calculate the definite integral between the two limits stated. In this case, between x = 1 and x = 3. Notationally we write

$$\int_{1}^{3} x^{2} + 1 \, \mathrm{d}x = \left[\frac{1}{3}x^{3} - x + C\right]_{1}^{3} = \left[g(x)\right]_{1}^{3} = g(3) - g(1) = \frac{32}{3}$$



Recommended links:

Highly recommended: HELM notes (Indefinite integrals: the basics, rules and table of standard results), HELM notes (Definite integrals: basics and engineering applications)

Recommended: Mathcentre handout (Explanation of integration as reverse of differentiation), Mathcentre handout (Explanation of integration as a summation, with engineering example), Mathcentre handout (Summary on definite integration) see also Mathcentre notes (Finding areas with integration)

Integration indef. & def. 2020