Integration of standard functions

Objectives:

- Learn how to find integrals of standard functions (as anti-derivatives)

Key points:

Integration methods are similar to differentiation. You either learn the integrals of popular functions or you can look them up in a table of standard integrals. Working backwards you can also use the table of standard derivatives to try and work out the integral of a function.

Example: We want to integrate $3\sin\left(x\pi+\frac{\pi}{4}\right)$ using variable x. Here are two typical options...

Option 1: Use a table of standard integrals and spot the matching pattern.

In a table of standard integrals it says that if $f(x)=\sin(\mathbf{a}x+\mathbf{b})$ then $\int f(x)\,\mathrm{d}x=-\frac{1}{\mathbf{a}}\cos(\mathbf{a}x+\mathbf{b})$. So by choosing $\mathbf{a}=\pi$ and $\mathbf{b}=\pi/4$ then

$$\int \sin\left(x\boldsymbol{\pi}+\frac{\boldsymbol{\pi}}{\mathbf{4}}\right)\,\mathrm{d}x=-\frac{1}{\boldsymbol{\pi}}\cos\left(x\boldsymbol{\pi}+\frac{\boldsymbol{\pi}}{\mathbf{4}}\right)+(\text{a constant})$$

But we started with $3\sin\left(x\pi+\frac{\pi}{4}\right)$ not $\sin\left(x\pi+\frac{\pi}{4}\right)$ and we remember that front constants can be copied unchanged, so the answer will be $-\frac{3}{\pi}\cos\left(x\pi+\frac{\pi}{4}\right)+(\text{a constant})$.

Option 2: Try and find a function whose derivative is our function.

The idea is to try to work out a function **whose derivative is** $3\sin\left(x\pi+\frac{\pi}{4}\right)$ this will be the integral of the function! We remember that differentiation and integration turn sin functions into cos functions and vice versa (along with maybe negative signs). So we know that to get $\sin\left(x\pi+\frac{\pi}{4}\right)$ we could first try... (using knowledge that the derivative of cos gives $-\sin$)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\cos\left(\boldsymbol{\pi}x+\frac{\boldsymbol{\pi}}{\mathbf{4}}\right)\right)=\boldsymbol{\pi}\times\left(-\sin\left(x\boldsymbol{\pi}+\frac{\boldsymbol{\pi}}{\mathbf{4}}\right)\right)=-\pi\sin\left(x\pi+\frac{\pi}{4}\right).$$

This is close, but the constant on the front is wrong, we need 3 not $-\pi$. But we are only searching, so we can just go back and multiply by $-\frac{3}{\pi}$ to start with! We therefore know that $-\frac{3}{\pi}\cos\left(\pi x+\frac{\pi}{4}\right)$ will differentiate into the function we started with – so this is the integral (if we add a constant).

Recommended links:

Highly recommended: Khan Academy mutlipart course (Rules, examples, follow course down to when it studies natural logarithm)

Other links: HELM handout (Just a table with lots of standard results, many not known at this stage)