Applications of integration

Objectives:

- Use integration to answer questions in engineering. e.g. total change; work down by a gas (a shaded graph areas); areas of objects; centroids.

Key points:

Integration can answer many engineering questions! The skill to try and learn is to **work out which function** and **what limits** to use (in a definite integral). Try lots of examples!

As a general trick: if you integrate over time, t, (i.e. so $\mathrm{d}t$ is on the end) then it is a bit like multiplying by time. So you **if you integrate a speed** (units m/s) **you will get a distance** (m) because you multiplied by time (s, seconds).

If you integrate a rate at which something happens over a time period then **you'll find out how much the thing changes over that period**. e.g. if $r(t) = 2e^{-0.3t}$ is the **rate of change of temperature** of a liquid at various times t then

$$\int_2^5 r(t)\,\mathrm{d}t = \Big[\text{actual change in temperature between time 2 and time 5}\Big].$$

More uses: the integral $\int_3^7 f(x)\,\mathrm{d}x$ equals the area between the curve of f and the x-axis between $x = 3$ and $x = 7$ (if the curve stays above). There's also a easy formula for centroids.

A popular example is to use integration to calculate 'work done' by a gas over a change in pressure and volume. For this, if you plot volume on the x-axis (litres), and pressure on the y-axis (atmospheres) then the area under the curve represents the 'work done' (in Joules).

There will alway be the integration to actually calculate at the end, which may still be difficult. However, this topic is more about learning what function to integrate, what range to integrate over (real-world examples are almost always definite integrals) and what the numerical answer means! The skills to actual do the integrations are elsewhere, including more at university!

Recommended links:

Highly recommended: Khan Academy multipart lesson (Good set of wordy modelling examples to follow)

Recommended: HELM handout (Integrating to find areas between curves)

Other links: Khan Academy example (Average value of a function, no need to follow proof parts), Khan Academy example (Advanced application to volume finding using cross-sections)