Static Characteristics of Signals

Noise

- Unwanted electrical signals that occur within measurement systems and that result in spurious signals occurring at the output are called noise.

- Noise can either be from extraneous sources [in which case it is generally called interference] or from within the system itself.

- Ultimately noise will limit the performance of the measuring system but various techniques can be employed to limit it to a greater or lesser extent.

Signal to Noise Ratio

- A signal is composed of two parts.
- signal = wanted signal + noise
- The signal to noise ratio is used to describe their relative magnitudes.
- S/N ratio usually quoted in decibels (dB).

Signal to Noise Ratio

$$\begin{aligned} S/N \text{ ratio} &= 10\log_{10}\frac{\text{signal power}}{\text{noise power}} \\ &= 20\log_{10}\frac{\text{signal voltage}}{\text{noise voltage}} \end{aligned}$$

Signal to Noise Ratio

For a system with and input and an output:

$$\text{Noise Ratio (NR)} = \frac{(S/N)_{in}}{(S/N)_{out}}$$

and

$$\text{Noise Figure (NF)} = 10\log_{10}\text{NR}$$

$$\Rightarrow 10\log_{10}\text{NR} = 10\log_{10}(S/N)_{\text{in}} - 10\log(S/N)_{out}$$

$$\Rightarrow (S/N)_{\text{out}}\ (\text{dB}) = (S/N)_{\text{in}}\ (\text{dB}) - \text{NF}$$

Signal Characterisation

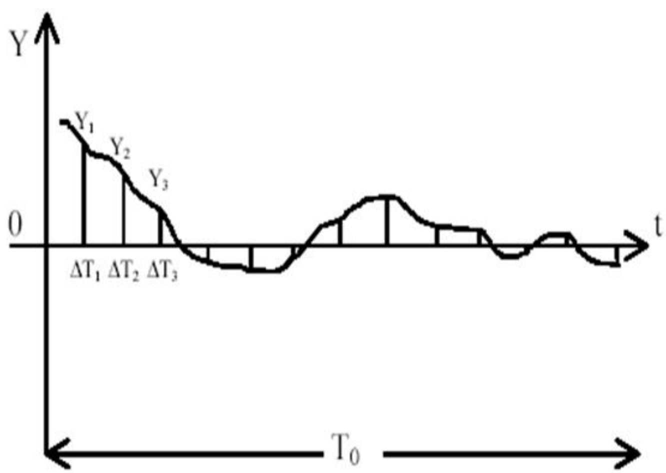
- Signal can be random or deterministic.
- Real signals from measurement systems are in general random, possibly with a deterministic component.
- Unwanted signals can also be random or deterministic.

Signal Characterisation

- Seven statistical quantities which can be used to estimate the behaviour of random signals.

  1. Mean
  2. Standard Deviation
  3. Skewness
  4. Kurtosis
  5. Probability Density Function
  6. Power Spectral Density
  7. Autocorrelation function

Signal Characterisation



- Take N samples of Y(t) at regular intervals T over a time period T_0.
- Assume:
  - T_0 long
  - N large.
  - Signal is stationary, i.e. long term statistical quantities do not vary with time.

Mean – 1st moment

1. <u>Mean</u> $\overline{y}$ for sample set is $\dfrac{1}{N}\sum_{i=1}^{N} y_i$

$$\lim \xrightarrow{N\to\infty,\ \Delta T\to 0} \frac{1}{T_0}\int_0^{T_0} y(t)dt \qquad \text{continuous signal}$$

Standard Deviation – 2nd moment

2. <u>Standard Deviation</u> σ_Y for sample: $\sigma_Y^2 = \frac{1}{N}\sum_{i=1}^{N}\left(Y_i - \overline{Y}\right)^2$

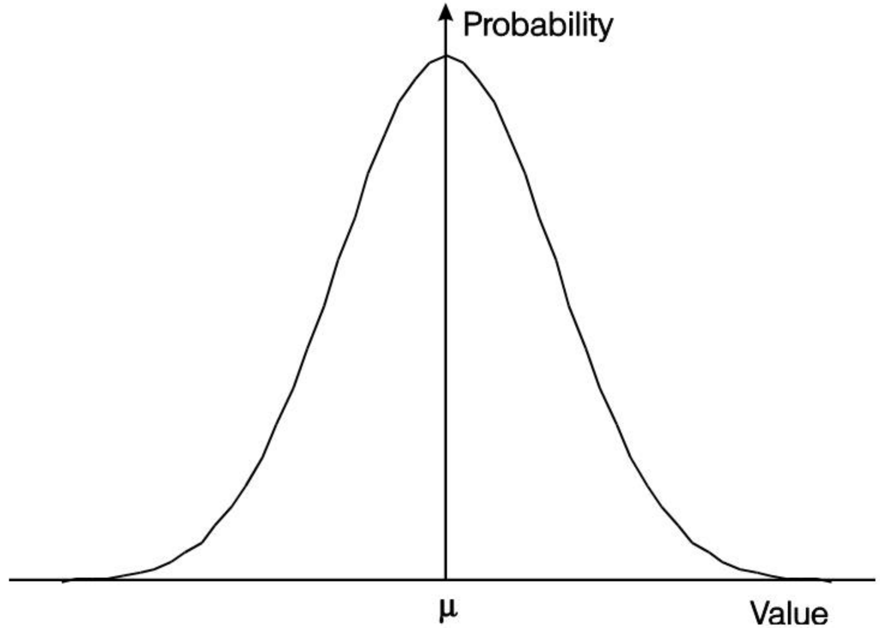
$$\lim \xrightarrow{N\to\infty,\ \Delta T\to 0} \sigma_Y^2 = \frac{1}{T_0}\int_0^{T_0}\left[y(t) - \overline{y}\right]^2 dt \quad \text{continuous signal}$$

Standard Deviation

Special Case, $\overline{y} = 0$: <u>standard deviation</u> σ is equal to the <u>root mean square</u> value y_{rms}.

$$y_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} y_i^2} \qquad \lim \xrightarrow{N\to\infty,\ \Delta T\to 0} \frac{1}{T_0}\int_0^{T_0} y^2(t)dt \qquad \{\text{for } \overline{y} = 0\}$$

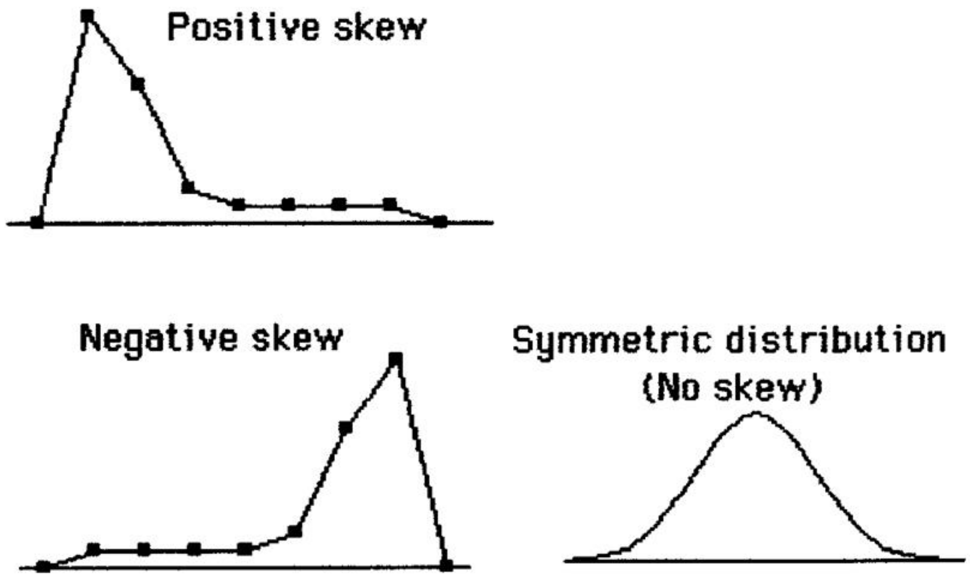
The normal distribution



Central Limit Theorem

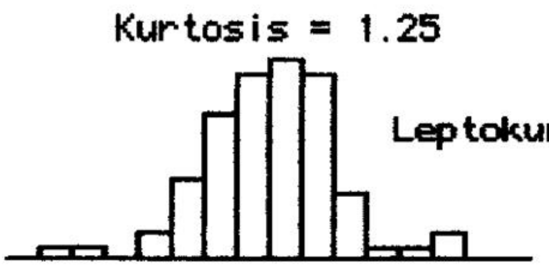
If a population of values has a finite standard deviation and mean then the distribution of sample means approaches the Normal Distribution with mean equal to and the standard deviation equal to S_N.

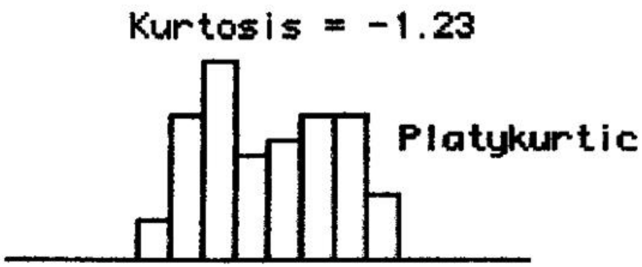
Skewness - 3rd Moment



$$\text{skewness} = \frac{\sum (x-\mu)^3}{N\sigma^3}$$

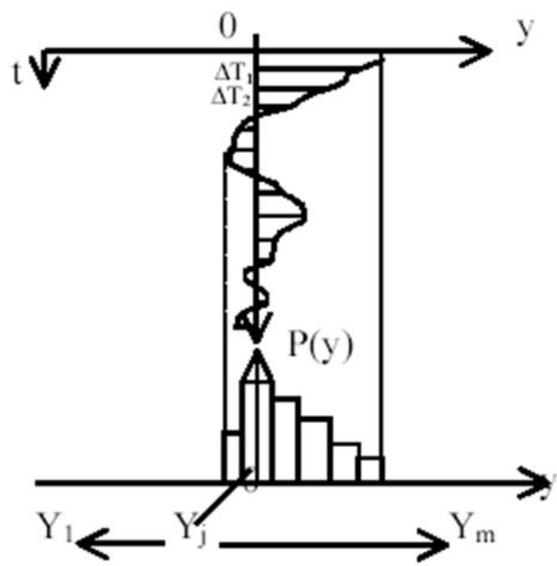
Kurtosis – 4th Moment





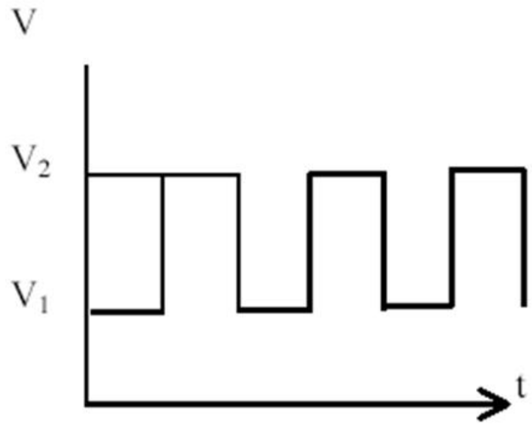
$$\text{kurtosis} = \frac{\sum (x-\mu)^4}{N\sigma^4} - 3$$

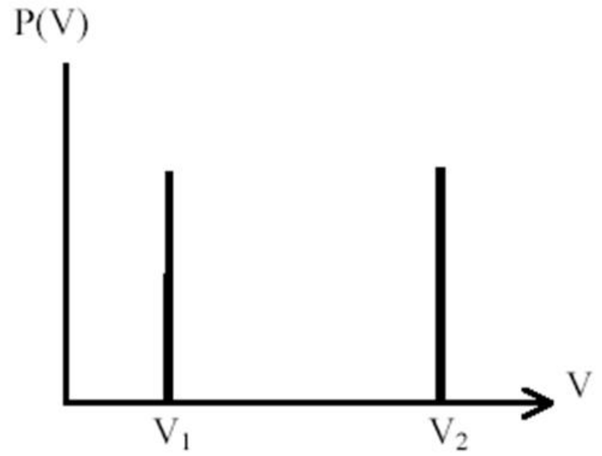
Probability Density Function



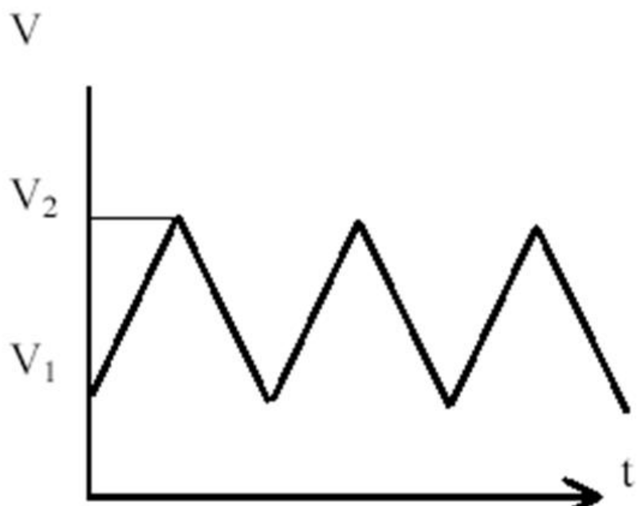
- Probability Density Function P(y) is the probability that a given sample lies in the interval $y \rightarrow y+dy$

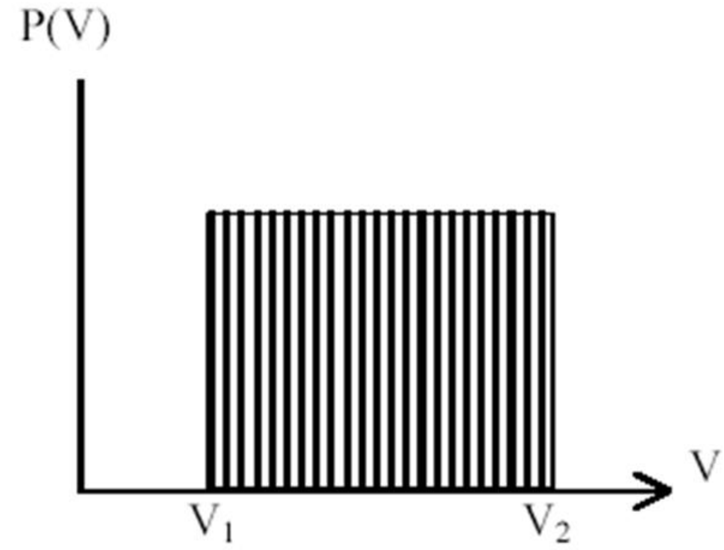
PDF – Square Wave



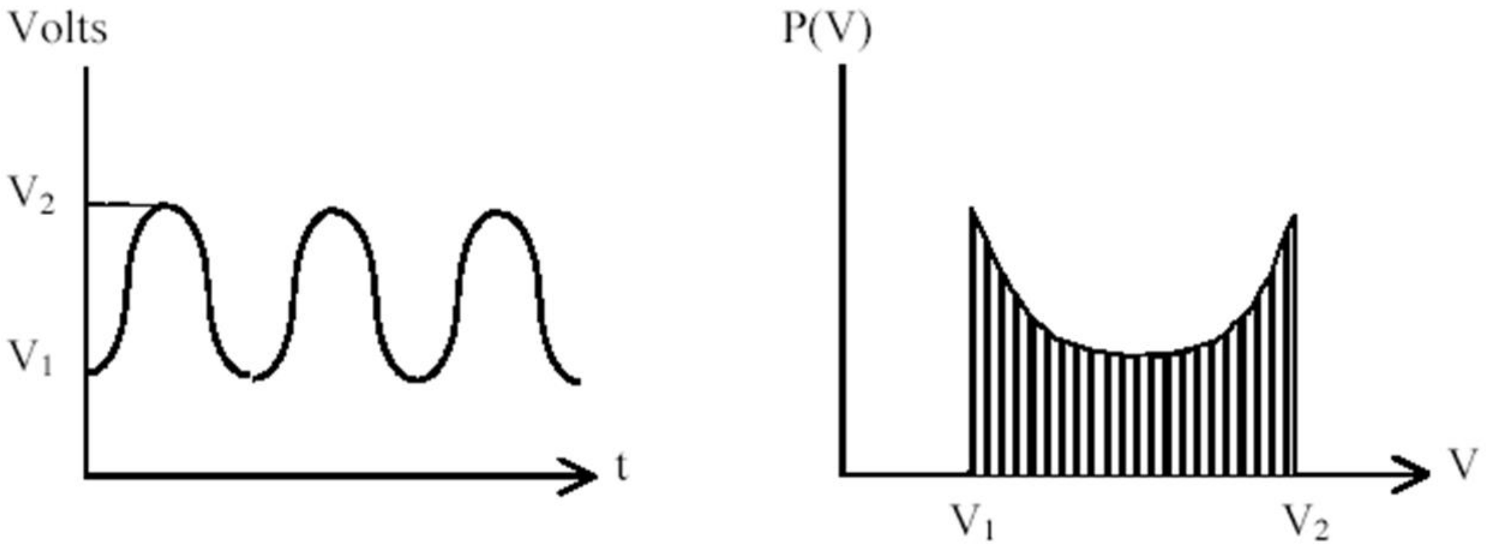


PDF – Triangular Wave

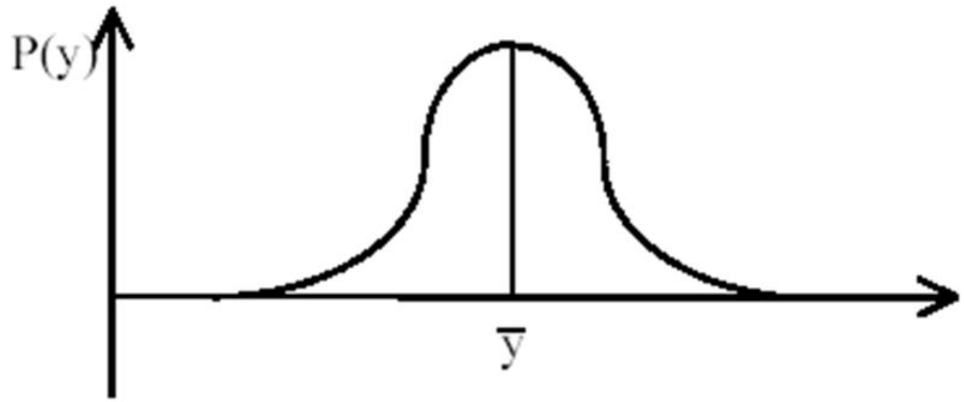




PDF – Sinusoidal Wave



PDF – Gaussian White Noise



$$P(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-(y - \overline{y})^2}{2\sigma^2}\right]$$

r.m.s. value is σ for $\overline{y} = 0$

Power Spectral Density

4. <u>Power Spectral Density</u>

Spectral Analysis: Example:- square wave.

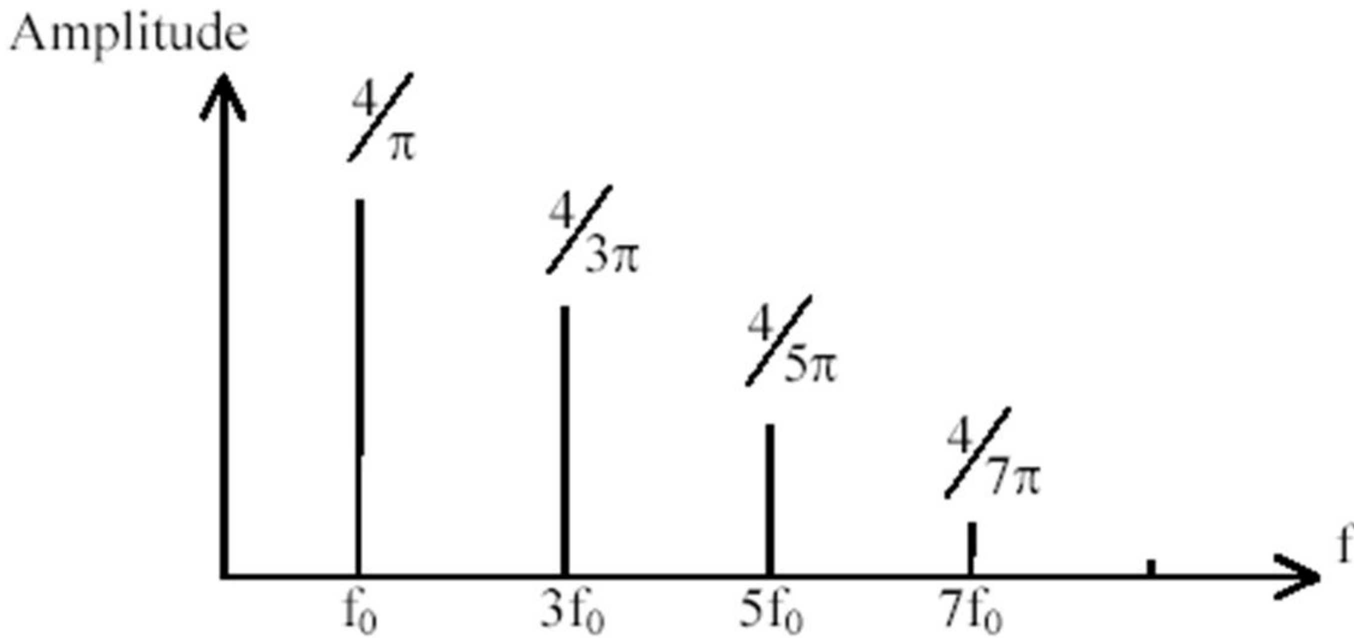
V(t)

Amplitude: 1 V, frequency f_0

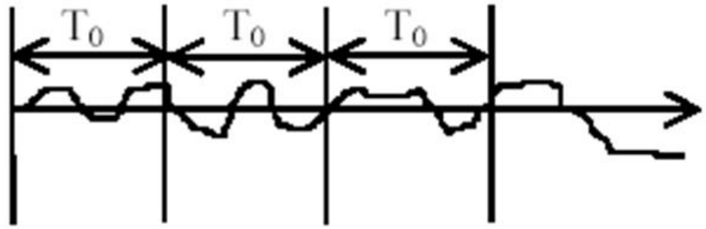
Can write V(t) as a Fourier series:

$$V(t) = \frac{4}{\pi}\sin 2\pi f_0 t + \frac{4}{3\pi}\sin 2\pi 3 f_0 t + \frac{4}{5\pi}\sin 2\pi 5 f_0 t + \frac{4}{7\pi}\sin 2\pi 7 f_0 t$$

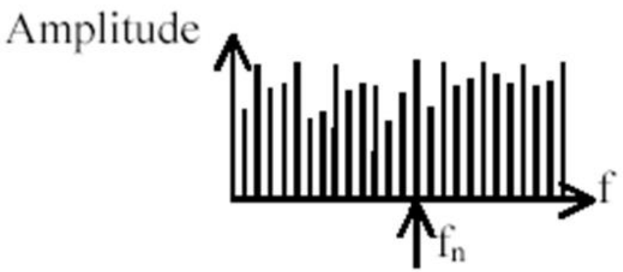
PSD for Square Wave



Power Spectral Density



Do Fourier analysis of successive samples of signal.



each time get a different amplitude and phase for component f_n.

Power Spectral Density

- Power that component f_n dissipates in a resistor R doesn't depend on phase.
- Average the power for f_n over several samples T_0 to produce <u>Power Density Spectrum</u>.

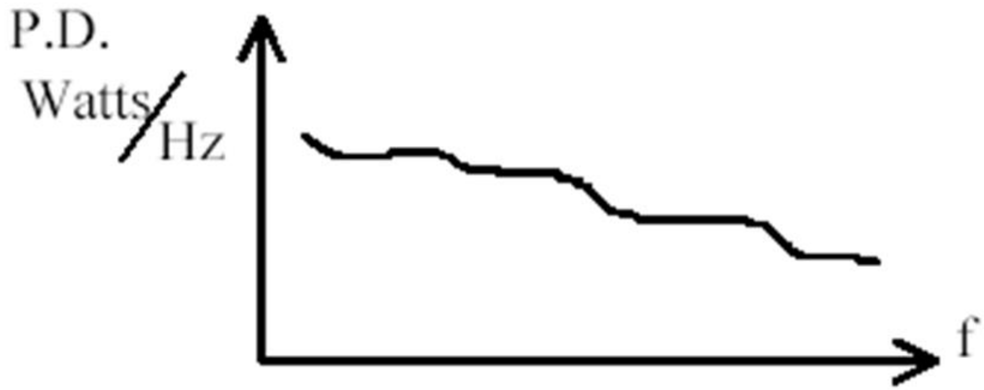
$$\Rightarrow P_n = V_{nrms}^2 / R$$

$$\text{Amplitude } A = \sqrt{2}\, V_{rms}$$

$$\Rightarrow P_n = \frac{A_n^2}{2R}$$

$\rightarrow$ now average power $P(f)$ over $f \rightarrow f + df$ to produce P.D.S.

Power Spectral Density



Autocorrelation Function

- The autocorrelation function is obtained by multiplying a signal with a delayed copy of itself.

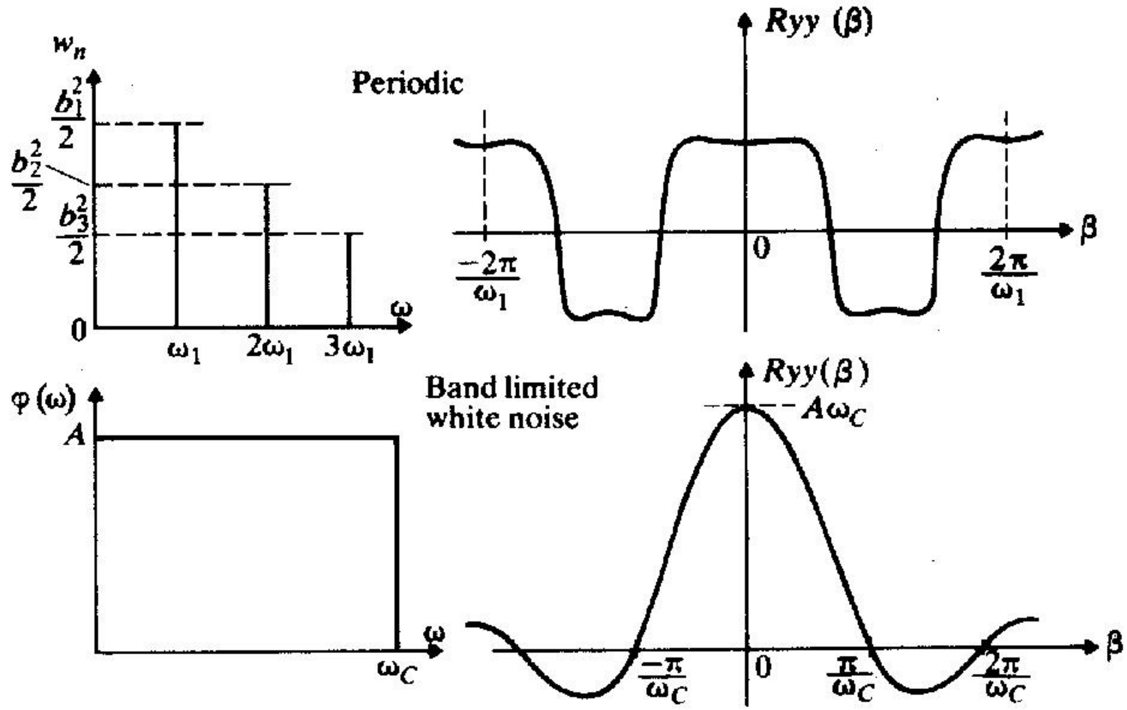
$$R_{yy}(\beta) = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_0^{T_0} y(t) y(t-\beta) dt$$

Autocorrelation Function

- For a sampled signal the autocorrelation function is given by:

$$R_{yy}(m\Delta T) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} y_i y_{i-m}$$

Autocorrelation Function



Autocorrelation function for periodic function and band limited white noise

Cyclic Averaging

- Method of noise reduction applicable to repetitive signals.
- The rms noise level σ_{av} of *p* averaged signals is reduced by

$$\sigma_{\mathrm{AV}} = \frac{\sigma}{\sqrt{p}}$$